The distribution of syntactic dependency distances

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ABSTRACT

The syntactic structure of a sentence can be represented as a graph, where vertices are words and edges indicate syntactic dependencies between them. In this setting, the distance between two linked words is defined as the difference between their positions. Here we wish to contribute to the characterization of the actual distribution of syntactic dependency distances, which has previously been argued to follow a power-law distribution. Here we propose a new model with two exponential regimes in which the probability decay is allowed to change after a break-point. This transition could mirror the transition from the processing of word chunks to higher-level structures. We find that a two-regime model – where the first regime follows either an exponential or a power-law decay – is the most likely one in all 20 languages we considered, independently of sentence length and annotation style. Moreover, the break-point exhibits low variation across languages and averages values of 4-5 words, suggesting that the amount of words that can be simultaneously processed abstracts from the specific language to a high degree. The probability decay slows down after the breakpoint, consistently with a universal chunk-and-pass mechanism. Finally, we give an account of the relation between the best estimated model and the closeness of syntactic dependencies as function of sentence length, according to a recently introduced optimality score.

Keywords: dependency syntax, dependency distance, exponential distribution, power-law distribution

1 Introduction

Language is one of the most complex and fascinating expressions of humans as social animals, stemming from our urge for communication and physical and cognitive limitations. The interaction between these two forces inevitably shapes language at many levels (Christiansen and Chater, 2016; Liu et al., 2017). Among them we here focus on syntax, namely the way in which words in a sentence compose into larger hierarchical structures, creating a parallel dimension to their plain linear arrangement. The hierarchical structure arises from the relations between words, modelled by means of a directed edge in the one-dimensional space of the network of a sentence (Figure 1). We call the resulting structure a syntactic dependency tree: each vertex is a word, and each word – besides the root – depends syntactically on its head, to which it is connected by an edge. We define d as the absolute value of the difference between the positions of two syntactically related words (Ferrer-i-Cancho, 2004). Thus, consecutive words are at distance 1, words separated by an intermediate word are at distance 2 and so on. For instance, in Figure 1 "John" and "gave" are at distance 1, "gave" and "painting" are at distance 3, and so on.

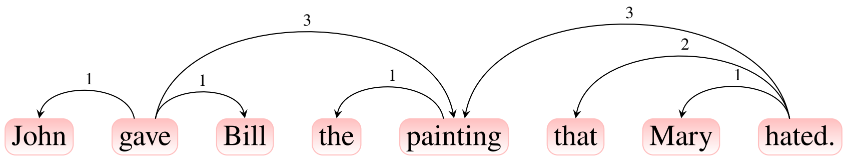


Figure 1: Example of syntactic dependency tree. Edges are labelled with the value of the syntactic dependency distance between the words they connect.

A well-established principle of Dependency Distance minimization (DDm) has been consistently found in languages, implying the preference for short dependencies (Ferrer-i-Cancho, 2004; Ferrer-i-Cancho et al., 2022; Futrell et al., 2015; Liu, 2008).

1.1 On the distribution of syntactic dependency distances

The large body of evidence in favor of DDm suggests that there are universal patterns underlying language structure, which are likely to reflect the functioning of the human brain rather than features of specific languages. Here we focus on the probability distribution of syntactic dependency distances as a window to that functioning (Liu et al., 2017). Ferrer-i-Cancho described the probability of a syntactic dependency as an exponentially decaying function of distance for sentences of fixed length in Czech and Romanian (Ferrer-i-Cancho, 2017; Ferrer-i-Cancho, 2004). However, he made an interesting observation concerning a change in the speed of the decay: the probability of observing a dependency at distance 4-5 or more is higher than expected, in the sense that the decay slows down, which apparently contradicts the DDm principle itself. Later on, Liu proposed a power-law behaviour to describe the distribution of dependency distances in a Chinese treebank, considering sentences of mixed length (Liu, 2007) that was later refined as a modified power law with an additional parameter (Liu, 2009). A later cross-linguistic study covering 30 languages identified a power-law distribution for long sentences, and an exponential trend in short ones (Lu and Liu, 2016). These approaches illustrate the complexity of the analysed problem. Nevertheless, all these distributions have a similar shape, characterized by the dominance of very short distances and a long tail (Jiang and Liu, 2015). The observed differences could hence derive from systematic discrepancies in sentence lengths, context, and annotation style, which all influence syntactic dependency distances (Ferrer-i-Cancho et al., 2022; Jiang and Liu, 2015). Moreover, power-laws can emerge from mixing other distributions, for instance from differently parameterized exponentials (Stumpf and Porter, 2012). Hence the need – expressed in various studies (Ferrer-i-Cancho, 2004; Ferrer-i-Cancho and Liu, 2014; Jiang and Liu, 2015) – to find the common ground of these results, analyzing the distribution of dependency distances while accounting for all these factors: considering both mixed and fixed sentence lengths in a large enough parallel corpus, while also controlling for annotation style.

1.2 Exponential distributions in nature

An exponential distribution of syntactic dependency distances was predicted assuming a constraint on the average distance between syntactically related words that was justified in terms of cognitive economy (Ferrer-i-Cancho, 2004). At a lower cognitive level, the exponential distribution of projection distances between cortical areas has been justified in terms of a general principle of wiring economy in neural networks (Ercsey-Ravasz et al., 2013).

It is worth framing our proposal of a two-regime exponential distribution for syntactic dependency distances in a broader setting where a breakpoint may indicate a boundary between local and non-local dynamics. A double exponential distribution for the average distance traversed by foraging ants is a robust phenomenon where the breakpoint separates risk-averse from risk-prone trajectories (Campos et al., 2016). A hypothesis for the origins of the breakpoint in the distribution of syntactic dependency distances is elaborated below.

1.3 Short-term memory (STM) limitations

Short-term memory (also called working memory), refers to a system, or a set of processes, holding mental representations temporarily available for use in thought and action (Cowan, 2017). G. Miller's classic article set the grounds for research on a possible absolute constraint on the amount of information that can be temporarily stored in memory, and on the mechanisms enacted to cope with it (Miller, 1956). The estimated values of this maximum span vary: 7 ± 2 (Miller, 1956), $2 - 3$ (Lewis and Vasishth, 2005) or 4 ± 1 (Cowan, 2001). However, it is commonly argued that such variation reflects variation in the unit of measurement: Miller's 7 ± 2 (Miller, 1956) would correspond to the amount of information before being compressed while lower values would correspond to chunks or compressed information (Mathy and Feldman, 2012).

These considerations on STM are particularly relevant in the scope of linguistic communication: communicating requires constantly receiving and processing new inputs, without losing reference to the previous ones. To illustrate this, suppose a left-to-right incremental processing of the sentence in Figure 1. Let an open dependency be one in which only one of the two elements that compose it has already appeared, and a closed dependency one in which both the head and the dependent have already been encountered. Then, in the context of dependency structure the success of communication depends on the ability to keep track of an open dependency while opening new ones, and without knowing *a priori* when it is going to be closed (Liu et al., 2017). Notice that dependencies represent relations between words, which are necessary for the speaker to convey a complex message building it from smaller units (encoding), and for the listener to recover such message by understanding the subjacent structure of the sequence of words (decoding). Thus, syntactic structure really reveals the way in which humans deal with physical limitations to be able to produce and process a potentially unbounded number of words. Christiansen and Chater provided an integrated framework to describe both the cognitive constraints affecting STM in language processing – what they call the "now-or-never bottleneck" – and the chunking strategy enacted to cope with them, which they refer to as "chunk-and-pass" mechanism (Christiansen and Chater, 2016). They collected a wide set of empirical results, describing the bottleneck as mainly arising from our short memory for auditory signals, the speed of new incoming linguistic input, and from memory limitations on sequence recalling tasks. According to the authors, to deal with these constraints the human cognitive system relies on a series of strategies. That is, as we receive new linguistic input, we eagerly process it by grouping units into chunks, and passing them at a more abstract level of representation; once a chunk has been integrated into the available knowledge hierarchy (Figure 1), a new one can be processed and again passed at higher representation levels. This model entails that chunking is required to store information for a longer time while a single word would be an easily forgotten piece of de-contextualized information, grouping words together produces a meaningful abstract image, which can be related to the following incoming concept. This mechanism would thus guarantee effective and efficient communication, profoundly shaping the structure of language itself.

1.4 Contribution

The primary aim of this work is to test the hypothesis that dependency distances in languages are distributed following two exponential regimes, modelled by means of a two-regime geometric distribution, and that the break-point between the regimes is similar across languages. The proposal of two regimes is motivated both empirically and theoretically. On one hand, it builds on the observations by Ferrer-i-Cancho concerning a change in probabilistic decay (Ferrer-i-Cancho, 2004). On the other hand, the existence of two different regimes would be consistent with the widely accepted idea that words are being chunked in order to be processed (Christiansen and Chater, 2016). Indeed, in a commentary on the work by Christiansen & Chater, Ferrer-i-Cancho had suggested a relation between his empirical observation and their processing framework, linking the chunking mechanism with the puzzling slowing down of probability decay in syntactic dependency distances after 4-5 words (Christiansen and Chater, 2016). Verifying this hypothesis opens the path for a deeper understanding of the distribution of syntactic dependency distances, and of how this could be influenced and shaped by universal constraints on memory. Concerning the first point, we believe our work will contribute to the existing literature on the distribution of dependency distances, finding a common ground to previous results by accounting for the effect of sentence length, context, and annotation style (Ferrer-i-Cancho, 2004; Ferrer-i-Cancho and Liu, 2014; Jiang and Liu, 2015). In fact, we consider both the syntactic structure of sentences with a specific length, and of various sentence lengths jointly, performing the analysis on a parallel corpus following two alternative syntactic dependency annotation schemes. The second point is related to one of the free parameters of our models, namely the break-point between the two regimes. If the change in probability is a mirror of the chunking mechanism enacted in language processing, the break-point we estimate could be a visible and direct statistical marker of the hypothesis advanced by Christiansen and Chater (2016). In particular, it may approximate the distance after which physical and cognitive limitations become too pressing, and the current chunk needs to be closed and encoded in memory, in order not to be overwritten by forthcoming information. Therefore, looking at the homogeneity of the estimated break-point values across languages could shed light on general cognitive patterns. Formally, we aim to verify the following two-fold hypothesis

- H_1. Syntactic dependency distances are distributed following two exponential regimes.
- H_2. The break-point between the two regimes exhibits low variation across languages and within a language.

Additionally, we further investigate the relation between the DDm principle and sentence length (Ferrer-i-Cancho and Gómez-Rodríguez, 2021), analysing how it is reflected in the shape of the distribution of syntactic dependency distances. We use Ω, a recently introduced optimality score, to quantify the intensity of DDm (Ferrer-i-Cancho et al., 2022).

1.5 Structure

The remainder of the article is organized as follows. In order to test H_1, we compare the fit of the proposed two-regime model against an ensemble of alternative distributions. Section 2 presents the definitions of the models for the distribution of syntactic dependency distances. Section 3 provides a detailed description of the data while Section 4 details the methodology. Section 5 reports the results of the model selection on sentences of languages from distinct families and investigates the relation between the best model and the optimality of syntactic dependency distances. Finally, section 6 discusses the findings, focusing on the verification of our hypotheses and on other general patterns while accounting for the observed cross-linguistic variability. Section 7 summarises the major conclusions of this article.

2 Models

We use $p(d)$ to refer to the probability that two linked words are at distance d. $d \in [1, n)$ in a sentence of n words. See Table 1 for a summary of the ensemble of models and Figure 2 for the shape of the models against an artificial random sample of their probability distributions (details on the generation of these samples are given in Appendix C). Here we present a series of well-known models (e.g., geometric distribution, right-truncated zeta distribution) and non-standard models for $p(d)$. The details of the derivation of the non-standard models are given in Appendix A.

Table 1: Models for the distribution of syntactic dependency distances. K is the number of free parameters. Refer to Appendix A for the derivation of the equations.

Model	Function	K	Definition
0	Null model	0	$\frac{1}{\binom{n}{2}}(n-d)$ if $d \in [1, n)$
0.0	Null model	1	$\frac{1}{\binom{d_{max}+1}{2}}(d_{max}+1-d)$ if $d \in [1, d_{max}]$
0.1	Extended Null model	0	$\sum_{n=min(n)}^{max(n)} \frac{n-d}{\binom{n}{2}} p(n)$ if $d \in [1, max(n))$
1	Geometric	1	$q(1-q)^{d-1}$ if $d \geq 1$
2	Right-truncated geometric	2	$\frac{q(1-q)^{d-1}}{1-(1-q)^{d_{max}}}$ if $d \in [1, d_{max}]$
3	Two-regime geometric	3	$\begin{cases} c_1(1-q_1)^{d-1} & \text{if } d \in [1, d_{max}] \\ c_2(1-q_2)^{d-1} & \text{if } d \geq d^* \end{cases}$
4	Two-regime - right-truncated geometric	4	$\begin{cases} c_1(1-q_1)^{d-1} & \text{if } d \in [1, d_{max}] \\ c_2(1-q_2)^{d-1} & \text{if } d \in [d^*, d_{max}] \end{cases}$
5	Right-truncated zeta distribution	2	$\frac{d^{-\gamma}}{H(d_{max}, \gamma)}$ if $d \geq 1$
6	Two-regime zeta-geometric	3	$\begin{cases} c_1 d^{-\gamma} & \text{if } d \in [1, d_{max}] \\ c_2(1-q)^{d-1} & \text{if } d \geq d^* \end{cases}$
7	Two-regime - right-truncated zeta-geometric	4	$\begin{cases} c_1 d^{-\gamma} & \text{if } d \in [1, d_{max}] \\ c_2(1-q)^{d-1} & \text{if } d \in [d^*, d_{max}] \end{cases}$

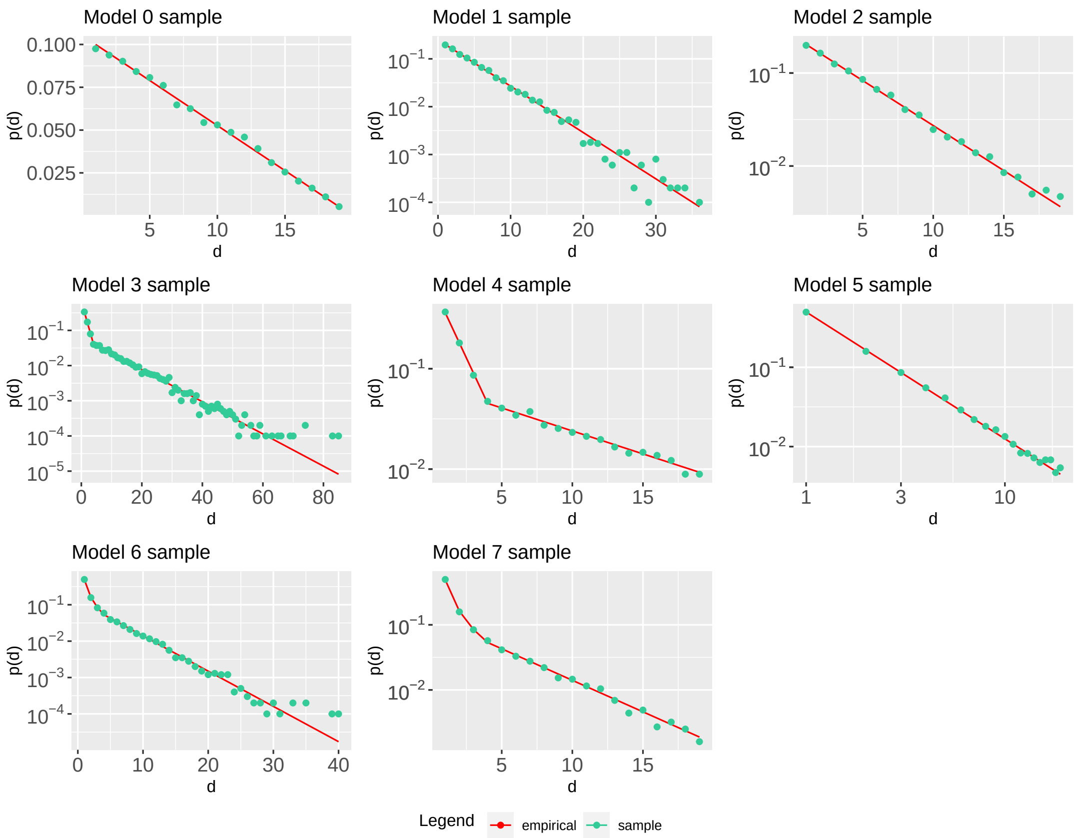


Figure 2: $p(d)$, the probability of d in a model versus a random sample of itself. The random sample has size 10^4. $n = 20$ ($d_{max} = 19$) for the right-truncated models. Thus Model 0 is the same as Model 0.0 here. $d^* = 4$ for the two-regime models. For the equations of the models refer to Table 1, while for the complete list of parameter values refer to Table 17.

The first model that we consider is Model 0, the null model obtained when a real sentence is shuffled at random or, equivalently, when there is no word order constraint (and all the $n!$ word orderings are equally likely). Then (Ferrer-i-Cancho, 2004)

$$p(d) = \begin{cases} \frac{1}{\binom{n}{2}}(n-d) & \text{if } d \in [1, n) \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The formulation of Model 0 in 1 assumes that the maximum distance is $n-1$ and that sentence length is unique, two assumptions that are too restrictive for our model selection setting. First, we do not know if actual maximum value of d is $n-1$ or a lower value that is unknown to us (but could be set by some memory limitations of the human brain). Second, we are interested in the best model by fixing sentence length (where sentence length is unique) and also when considering jointly all sentences of any length for a given language (where sentence length varies). Thus, for fitting purposes, we distinguish between two specifications of Model 0. In the first one, Model 0.0, we relax the first assumption and give the model the freedom to select a maximum distance that does not need to be $n-1$, the theoretical maximum value of d. Accordingly, Model 0.0 is defined as

$$p(d) = \begin{cases} \frac{1}{\binom{d_{max}+1}{2}}(d_{max} + 1 - d) & \text{if } d \in [1, d_{max}] \\ 0 & \text{otherwise} \end{cases}$$

where d_{max} is the only free parameter. The second specification of Model 0, Model 0.1 adapts the initial Model 0 to sentences of mixed lengths. Suppose that $p(n)$ is the proportion of sentences having length n, and $\min(n)$ and $max(n)$ are the minimum and maximum observed values of n in the sample. Then Model 0.1 is defined as

$$p(d) = \begin{cases} \sum_{n=min(n)}^{max(n)} \frac{n-d}{\binom{n}{2}} p(n) & \text{if } d \in [1, max(n)) \\ 0 & \text{otherwise.} \end{cases}$$

The following models follow the same design principle of Model 0.0 and, for the sake of simplicity, do not introduce n into the definition of the model as Model 0 or Model 0.1.

Given that distances are discrete, an exponential decay can be modeled with a geometric curve. Thus, Model 1 is the displaced geometric distribution, defined as

$$p(d) = \begin{cases} q(1-q)^{d-1} & \text{if } d \geq 1 \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where q is the only free parameter. When $d \geq n$, the displaced geometric assumes that $p(d) > 0$ while in a real sentence $p(d) = 0$. For this reason, we also consider Model 2, that is a right-truncated version in which non-zero probability mass is restricted to $d \in [1, d_{max}]$, i.e.

$$p(d) = \begin{cases} \frac{q(1-q)^{d-1}}{1-(1-q)^{d_{max}}} & \text{if } d \in [1, d_{max}) \\ 0 & \text{otherwise,} \end{cases}$$

The two-regime models are obtained by splitting the range of variation of d into two overlapping regimes, one for $1 \leq d \leq d^*$ and another for $d \geq d^*$, where $p'(d)$ and $p''(d)$, the probability mass in the first and in the second regime respectively, satisfy $p'(d^*) = p''(d^*)$. Accordingly, Model 3 is a generalization of Model 1 that consists of two regimes, and is defined as

$$p(d) = \begin{cases} c_1(1-q_1)^{d-1} & \text{if } d \in [1, d^*] \\ c_2(1-q_2)^{d-1} & \text{if } d \geq d^* \\ 0 & \text{otherwise,} \end{cases}$$

where c_1 and c_2 are normalization factors defined as

$$c_1 = \frac{q_1 q_2}{q_2 + (1-q_1)^{d^*-1}(q_1 - q_2)} \tag{3}$$

$$c_2 = \tau c_1$$

$$\tau = \frac{(1-q_1)^{d^*-1}}{(1-q_2)^{d^*-1}}. \tag{4}$$

Thus, the only free parameters of Model 3 are q_1, q_2 and d^*.

Model 4 is a generalization of Model 3 by right truncation, that is

$$p(d) = \begin{cases} c_1(1-q_1)^{d-1} & \text{if } d \in [1, d^*] \\ c_2(1-q_2)^{d-1} & \text{if } d \in [d^*, d_{max}], \\ 0 & \text{otherwise,} \end{cases}$$

where c_1 and c_2 are normalization factors defined as

$$c_1 = \frac{q_1 q_2}{q_2 + (1-q_1)^{d^*-1}(q_1 - q_2 - q_1(1-q_2)^{d_{max}-d^*+1})}. \tag{5}$$

and $c_2 = \tau c_1$ with τ defined as in 4. The only free parameters of Model 4 are q_1, q_2, d^* and d_{max}.

Next, following previous on syntactic dependency distances (Liu, 2007), we also consider Model 5, a power-law model that is a right-truncated zeta distribution with parameters γ and d_{max} (Wimmer and Altmann, 1999), that is defined as follows

$$p(d) = \begin{cases} \frac{d^{-\gamma}}{H(d_{max},\gamma)} & \text{if } d \geq 1 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$H(d_{max}, \gamma) = \sum_{k=1}^{d_{max}} \frac{1}{k^\gamma}$$

is the generalized harmonic number of order γ of d_{max}. Finally, we introduce Models 6 and 7, that are also composed of two regimes, the first one distributed as a right-truncated power-law and the second one as a geometric curve. Model 6 is defined as

$$p(d) = \begin{cases} c_1 d^{-\gamma} & \text{if } d \in [1, d^*] \\ c_2(1-q)^{d-1} & \text{if } d \geq d^* \\ 0 & \text{otherwise,} \end{cases}$$

where c_1 and c_2 are normalization factors defined as

$$c_1 = \frac{q}{qH(d^*, \gamma) + d^{*-\gamma}(1-q)} \tag{6}$$

$$c_2 = \tau c_1$$

$$\tau = \frac{d^{*^{-\gamma}}}{(1-q)^{d^*-1}}. \tag{7}$$

Model 7, the right-truncated version of Model 6, is defined as

$$p(d) = \begin{cases} c_1 d^{-\gamma} & \text{if } d \in [1, d^*] \\ c_2(1-q)^{d-1} & \text{if } d \in [d^*, d_{max}] \\ 0 & \text{otherwise,} \end{cases}$$

where

$$c_1 = \frac{q}{qH(d^*, \gamma) + d^{*-\gamma}(1-q-(1-q)^{d_{max}-d^*+1})}, \tag{8}$$

and $c_2 = \tau c_1$ with τ defined as in 7. The only free parameters of Model 6 are γ, d^* and q. Model 7 adds a third free parameter that is d_{max}.

2.1 Speed of decay

When plotted in log-linear scale, an exponential curve becomes a line. For a geometric model (2), the slope of that line is $\log(1-q)$ since

$$\begin{aligned}\log p(d) &= \log q(1-q)^{d-1} \\ &= d\log(1-q) + \log\frac{q}{1-q}.\end{aligned}$$

That slope conveys information about the speed of probability decay. Such slope is a decreasing function of q (Figure 3), meaning that as q increases the slope becomes more negative, and probability decays faster. In light of this fact, we consider parameters q (Models 1 and 2) as well as q_1 and q_2 (Models 3-4) to account for the speed of exponential decay in the two regimes of Models 3-4, and we refer to them as "slope parameters" for simplicity.

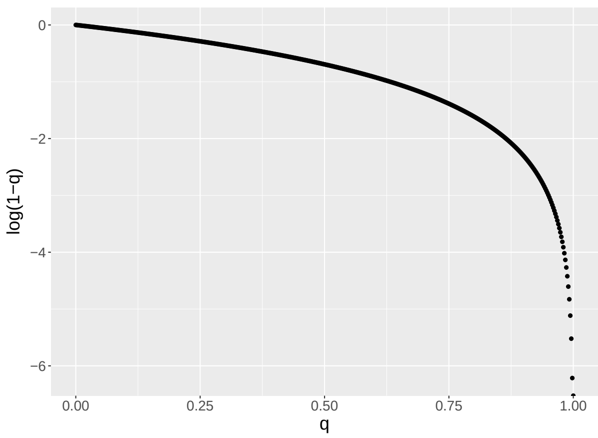


Figure 3: Slope of a geometric curve in log-linear scale as a function of its parameter q for $q \in [0, 1)$.

3 Material

Table 2: The languages, their linguistic family and their writing system.

Language	Family	Writing system
Arabic	Afro-Asiatic	Arabic
Chinese	Sino-Tibetan	Han
Czech	Indo-European	Latin
English	Indo-European	Latin
Finnish	Uralic	Latin
French	Indo-European	Latin
German	Indo-European	Latin
Hindi	Indo-European	Devanagari
Icelandic	Indo-European	Latin
Indonesian	Austronesian	Latin
Italian	Indo-European	Latin
Japanese	Japonic	Japanese
Korean	Koreanic	Hangul
Polish	Indo-European	Latin
Portuguese	Indo-European	Latin
Russian	Indo-European	Cyrillic
Spanish	Indo-European	Latin
Swedish	Indo-European	Latin
Thai	Kra-Dai	Thai
Turkish	Turkic	Latin

We extract syntactic dependency distances from a parallel subset of 20 languages from the Universal Dependencies collection (Nivre et al., 2017). See Table 2 for the languages, their linguistic family and their writing system. This subset is parallel in the sense that it contains the same sentences translated into every language. We use version 2.6, available here. Parallelism is crucial for robust cross-linguistic comparisons, as context can largely influence various aspects of language, including dependency structure. Another factor that shall be considered is annotation style, as there is no univocal way to generate syntactic dependency trees starting from a sentence. For this reason, we compare two different annotation styles: Universal Dependencies (Nivre et al., 2017) and the alternative Surface Syntactic Universal Dependencies (Gerdes et al., 2018). We refer to the two resulting versions of the collection as PUD and PSUD. See Table 3 and Table 4 for a summary of the main statistical features of PUD and PSUD respectively. It can be seen that mean dependency distance values $(\mathrm{mean}(d))$ are smaller in PSUD.

Table 3: Summary of PUD collection. $\#s$ stands for number of sentences, $\#d$ stands for number of distances.

Language	$\#s$	$\#d$	$\min(d)$	$\text{mean}(d)$	$\max(d)$	$\min(n)$	$\text{mean}(n)$	$\max(n)$
Arabic	995	17514	1	2.30	30	3	18.60	50
Czech	995	14976	1	2.39	29	3	16.05	44
German	995	17544	1	3.11	42	4	18.63	50
English	995	17711	1	2.53	31	4	18.80	56
Finnish	995	12465	1	2.24	21	3	13.53	39
French	995	21165	1	2.52	36	4	22.27	54
Hindi	995	20517	1	3.30	42	4	21.62	58
Indonesian	995	16311	1	2.26	27	3	17.39	47
Icelandic	995	15860	1	2.32	34	3	16.94	52
Italian	995	20413	1	2.48	35	3	21.52	60
Japanese	995	24703	1	2.97	65	4	25.83	70
Korean	995	13978	1	2.75	37	3	15.05	43
Polish	995	14720	1	2.23	27	3	15.79	39
Portuguese	995	19808	1	2.53	34	4	20.91	58
Russian	995	15369	1	2.27	32	3	16.45	47
Spanish	995	19986	1	2.50	32	3	21.09	58
Swedish	995	16119	1	2.47	31	4	17.20	49
Thai	995	21034	1	2.44	38	4	22.14	63
Turkish	995	13727	1	2.91	34	3	14.80	37
Chinese	995	17501	1	3.09	39	3	18.59	49

Table 4: Summary of PSUD collection. $\#s$ stands for number of sentences, $\#d$ stands for number of distances.

Language	$\#s$	$\#d$	$\min(d)$	$\text{mean}(d)$	$\max(d)$	$\min(n)$	$\text{mean}(n)$	$\max(n)$
Arabic	995	17514	1	2.05	30	3	18.60	50
Czech	995	14976	1	2.11	29	3	16.05	44
German	995	17544	1	2.82	38	4	18.63	50
English	995	17711	1	2.12	31	4	18.80	56
Finnish	995	12465	1	2.04	22	3	13.53	39
French	995	21165	1	2.13	35	4	22.27	54
Hindi	995	20517	1	3.04	38	4	21.62	58
Indonesian	995	16311	1	2.00	27	3	17.39	47
Icelandic	995	15860	1	1.92	34	3	16.94	52
Italian	995	20413	1	2.10	35	3	21.52	60
Japanese	995	24703	1	2.73	67	4	25.83	70
Korean	995	13978	1	2.70	38	3	15.05	43
Polish	995	14720	1	2.00	27	3	15.79	39
Portuguese	995	19808	1	2.13	34	4	20.91	58
Russian	995	15369	1	2.05	32	3	16.45	47
Spanish	995	19986	1	2.13	31	3	21.09	58
Swedish	995	16119	1	2.07	31	4	17.20	49
Thai	995	21034	1	2.20	39	4	22.14	63
Turkish	995	13727	1	2.86	33	3	14.80	37
Chinese	995	17501	1	2.99	39	3	18.59	49

4 Methodology

The code for this work was written both in `R` and `python`, and is available here.

4.1 Model selection

We here describe the model selection procedure implemented to test H_1. This methodology is validated with the help of artificially generated random samples from a given distribution (Appendix C).

Optimal parameters for each model are estimated by maximum likelihood. Then, the best model is selected according to Information Criteria (Anderson and Burnham, 2004). In real languages (this section), models are compared through Akaike Information Criterion (AIC). In artificially generated random samples (Appendix C), the best model is better selected through Bayes Information Criterion (BIC) because the true data generating process is known. BIC differs from AIC by relying on the assumption that the real distribution is among the tested ones (Wagenmakers and Farrell, 2004). For a given model, we use the following definitions of these scores (Anderson and Burnham, 2004)

$$\begin{aligned} AIC &= -2\mathcal{L} + 2K\frac{K}{N-K-1} \\ BIC &= -2\mathcal{L} + K\log N, \end{aligned} \tag{9}$$

where K is the number of parameters of the model and N is the sample size. With respect to AIC, the criterion proposed by Schwarz (BIC) applies a stronger penalty for the number of parameters.

Given that both AIC and BIC are measures of information loss, the best model for a sample is the one minimizing the selected score. We aim to find the best model for a sample of N distances $\{d_1, d_2, ..., d_i, ..., d_N\}$, where $\min(d)$ and $\max(d)$ are, respectively, the minimum and maximum observed distances, and $f(d)$ is the frequency of distance d in the sample. Then the sample size is

$$N = \sum_{i=1}^{max(d)} f(d_i) = \sum_{d=1}^{max(d)} f(d).$$

The log-likelihood functions of the models are summarized in Table 13. See Appendix B for a derivation of the log-likelihood functions for each model.

4.1.1 Parameter estimation

Maximum likelihood estimation (MLE) algorithms require one to specify the range of variation of the parameters as well as proper initial values. It is well-known that MLE methods are highly sensitive to the choice of the starting values, as they may incur local optima when minimizing the minus log-likelihood function (Myung, 2003). Here we explain the criteria used to select the initial value and the range of variation of the parameters, which are summarised in Table 5 and Table 6 respectively. Let x_{init} be the initial value of parameter x. Also, let $\max_i(d)$ be the $i-th$ largest value of d in the sample, so that $\max_1(d) = max(d)$. Similarly, let $\min_i(d)$ be the $i-th$ smallest value of d in the sample, so that $\min_1(d) = min(d)$.

Table 5: The initial values of the parameters for maximum likelihood estimation. Here Model 0 refers to model 0.0.

Model	d_{max}	q	q_1	q_2	d^*	γ
0	$\max(d)$	-	-	-	-	-
1	-	q_{init}	-	-	-	-
2	$\max(d)$	q_{init}	-	-	-	-
3	-	-	q_{1init}	q_{2init}	5	-
4	$\max(d)$	-	q_{1init}	q_{2init}	5	-
5	$\max(d)$	-	-	-	-	γ_{init}
6	-	q_{init}	-	-	5	γ_{init}
7	$\max(d)$	q_{init}	-	-	5	γ_{init}

The rationale for the choices in Table 5 and Table 6 is as follows

- d_{max}. The maximum observed distance is both the starting point and smallest admissible value, while there is no upper bound.
- q. In the geometric models (Models 1 and 2), the initial value for q, q_{1init}, is the maximum likelihood estimator, i.e. the inverse of the mean observed distance $q_{init} = 1/mean(d)$. The bounds are set so that $q \in (0, 1)$ to avoid values out of the domain of the log-likelihood function. In Models 6 and 7, the initial value of q for the second regime is set to the maximum likelihood estimator $1/mean(d)$ of an ideal geometric distribution, but restricting the mean to distances greater than d^*.
- q_1 and q_2. These two parameters are both initialized by first running a linear regression on $\log p(d)$ and d, for $d \leq d^*$ in the case of q_{1init}, and for $d \geq d^*$ in the case of q_{2init}. Then, the respective slopes β_1 and β_2 are used to compute the initial values via $q_{1init} = 1 - e^{\beta_1}$ and $q_{2init} = 1 - e^{\beta_2}$. Notice that, as the tail of the distribution is noisy, the estimated slope sometimes results in a 0 or even a positive value for values of d^* very close to $\max(d)$. When that happened, the corresponding q_{2init} was set to its lower bound. As in q, the bounds are set so that $q_1, q_2 \in (0, 1)$.
- d^*. The initial value is 5, as suggested by the visual inspection of the plots. The parameter is bounded to vary between $\min_2(d)$ and $\max_2(d)$, based on the minimum requirement on the size of the two regimes (section 4.1.3). Indeed, by setting d^* to either $\min_1(d)$ or to $\max_1(d)$, one of the two regimes would only be composed by one isolated observation, from which no trend can be inferred. Incidentally, the DDm principle, predicts that $\min_2(d) = 2$ if n is large enough (Ferrer-i-Cancho, 2004).
- γ. For Model 5, the initial value of the MLE estimator of the exponent of a continuous power-law (Newman, 2005):

$$\gamma_{init} = 1 + N \left[\sum_{i=1}^{N} \frac{d_i}{min(d)} \right]^{-1}.$$

For Models 6 and 7 (where only the first regime follows a zeta distribution), γ_{init} is computed over the distances up to d^*.

Table 6: The lower (low) and upper (up) bounds of the parameters for maximum likelihood estimation. $\epsilon = 10^{-3}$. Here Model 0 refers to Model 0.0.

	d_{max}		q		q_1		q_2		d^*		γ	
Model	**low**	**up**	**low**	**up**	**low**	**up**	**low**	**up**	**low**	**up**	**low**	**up**
0	$\max(d)$	∞	-	-	-	-	-	-	-	-	-	-
1	-	-	ϵ	$1-\epsilon$	-	-	-	-	-	-	-	-
2	$\max(d)$	∞	ϵ	$1-\epsilon$	-	-	-	-	-	-	-	-
3	-	-	-	-	ϵ	$1-\epsilon$	ϵ	$1-\epsilon$	$\min_2(d)$	$\max_2(d)$	-	-
4	$\max(d)$	∞	-	-	ϵ	$1-\epsilon$	ϵ	$1-\epsilon$	$\min_2(d)$	$\max_2(d)$	-	-
5	$\max(d)$	∞	-	-	-	-	-	-	-	-	0	∞
6	-	-	ϵ	$1-\epsilon$	-	-	-	-	$\min_2(d)$	$\max_2(d)$	0	∞
7	$\max(d)$	∞	ϵ	$1-\epsilon$	-	-	-	-	$\min_2(d)$	$\max_2(d)$	0	∞

4.1.2 Maximum likelihood estimation (MLE)

We considered two MLE methods in `R`: `mle()` from `stats2` and `mle2()` from the `bbmle` package (Bolker, 2007). The base `R` implementation, `mle()`, may explore values out of the specified bounds thus resulting in errors. Where this is the case, we resort to the enhanced, more robust version of the optimizer, `mle2()`, which is able to return a result even if the algorithm does not reach convergence. Both `mle2a()` and `mle2()` optimize on a continuous space. Hence, for the discrete parameters, i.e. d^* and d_{max}, we retrieved their most likely value by exhaustively exploring all values included between their theoretical bounds. In this way, we also decrease the complexity of MLE by reducing the number of parameters to be optimized through the call to `mle()` or `mle2()`. Thus, for each value of d^* (and d_{max} in the right-truncated models) we optimized the remaining parameters, and finally selected the parameters combination resulting in the highest log-likelihood.

4.1.3 Requirements for two-regime models

In order to fit a double-regime model to a data sample, we need $N \geq 3$. In fact, at least two points are needed in order to infer a speed of probability decay within a regime, meaning that each regime has to contain at least 2 distinct observations. Given that the value assigned to the break-point is common to the two regimes, this results in a requirement of $N \geq 3$. See Figure 4 for an example of this scenario, displaying the distribution of syntactic dependency distances for sentences of 4 words in Italian, annotated according to SUD. Notice that this requirement directly implies that sentences with $n < 4$ are excluded from the model selection procedure.

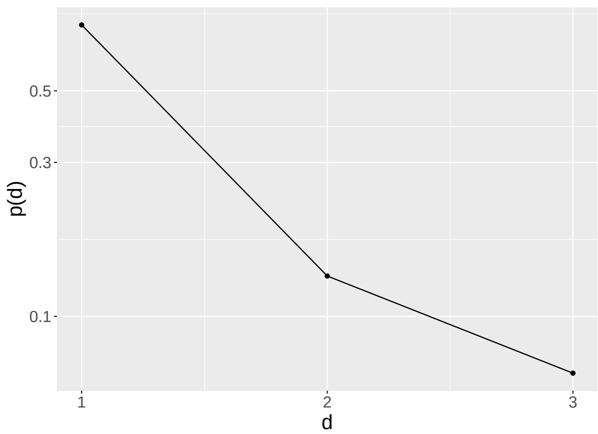


Figure 4: Syntactic dependency distance distribution of sentences with 4 words in Italian, annotated according to SUD. Only three unique values of d have been observed.

4.1.4 Representativeness

When performing model selection on sentences of specific lengths from a certain language, we obtain a set of best models, one for each sentence length. To summarize this information and obtain a single best model for each language, we consider the most frequent best model within that language. However, this raises the concern of whether all best models are equally reliable, as some of them are estimated on a single sentence. For instance, very long sentences, which are normally rare, are thus likely to be underrepresented in the data. On the other hand, setting a single specific threshold on the minimum number of sentences required for a sentence length to be included in the voting procedure would mistakenly hide important aspects of the analysis. In fact, the suitable threshold should depend on sentence length itself. Consider a very long sentence, composed of 50 words, and a very short one, of only 4 words. While – keeping fixed the syntactic structure – the first one could appear with $50!$ different re-orderings, the second one could only be written in $4!$ possible ways. Thus, a single sentence observed for $n = 4$ is much more representative (as the expected variability in dependency distance is lower) for the whole length category than a single one observed for $n = 50$. For this reason, we report the most frequent best models both when no threshold is set (Table 8) and for increasing representativeness threshold (Figure 7).

4.2 The Ω optimality score

Ω is a recently introduced optimality score for the closeness of syntactic dependency distances, which integrated normalization with respect to both a minimum and a random baseline (Ferrer-i-Cancho et al., 2022). The score is defined as

$$\Omega = \frac{D_{rla} - D}{D_{rla} - D_{min}},$$

where D is the observed sum of dependency distances in a sentence, D_{rla} is the expected sum of dependency distances in a uniformly random linear arrangement of its words (Ferrer-i-Cancho, 2004, 2019), i.e.

$$D_{rla} = \frac{n^2 - 1}{3} \tag{10}$$

and D_{min} is the sum of dependency distances in a minimum linear arrangement of the words (Esteban and Ferrer-i-Cancho, 2017; Shiloach, 1979). Both baselines assume that the network structure is fixed. D and D_{min} are computed using the `python` interface of the Linear Arrangement Library (Alemany-Puig et al., 2021).

Positive values of Ω indicate that syntactic dependency distances in the sentence are shorter than one would expect from picking uniformly at random among all the possible $n!$ orderings. The maximum, $\Omega = 1$, is reached when $D = D_{min}$. Conversely, a negative value indicates that distances are being maximized, as they are higher than expected in a random shuffling of words in a sentence. When word order is random, Ω will take values tending to 0. $\langle\Omega\rangle$ is the average value of Ω over individual sentences.

5 Results

We fit the models introduced in the Section 2 to a parallel collection of texts from 20 languages called PUD, that has been annotated with syntactic dependencies as in Figure 1. To control for annotation style we consider two variants, PUD with the original annotation style (Nivre et al., 2017) and PSUD, that follows the alternative SUD annotation style (Gerdes et al., 2018). Refer to section 3 for further details on the data, and to section 4.1 for a complete description of the model selection procedure.

This section is organized as follows. First, we report on the best models (Section 5.1), the break-points of the two-regime models (Section 5.2) and the relationship between slope parameters (q_1 and q_2) for each language (Section 5.3), both by considering fixed and mixed sentence lengths. We define representativeness threshold, shortly representativeness, as the minimum number of distinct sentences with a certain length for such length to be included in model selection (a further justification of this threshold is found in Section 4.1.4. Section 5.1 investigates the robustness of conclusions with respect to sample representativeness. Detailed tables of the estimated parameters, Akaike Information Criterion (AIC) scores, and AIC differences for both collections can be found in Appendix D. Second, we will investigate the relationship between the best model and the degree of optimality of syntactic dependency distances on sentences of fixed length (Section 5.4. Notice that we often refer jointly to Models 3 and 4 (6 and 7) as 3-4 (6-7), given that they model the same probability distribution with or without a right-truncation point.

5.1 Model selection

The best model to describe syntactic dependency distances independent of sentence length is composed of two regimes in every language and collection (Table 7). Models 3-4 dominate over Models 6-7, with 13/20 languages in PUD and 11/20 in PSUD having Model 3 or 4 as the best one. We find overall agreement between the two annotation styles, both in terms of best model and in terms of right-truncation. The exceptions to this agreement are Indonesian and Japanese – for which PUD yields an exponential decay in the first regime, while PSUD identifies a power-law one – and Chinese, English, and Italian, where the best model in PUD and PSUD differs by right truncation. In Figure 5, we show how the best models in PUD are able to accurately capture the bulk of the distribution, with some variability left in the tail. The equivalent figure for PSUD can be found in Appendix D.

Table 7: Best model for the distribution of syntactic dependency distances in sentences of mixed lengths for every language and collection. Models 3-4 are marked with pink and Models 6-7 with blue to ease visualization.

Language	PUD	PSUD
Arabic	7	7
Chinese	6	7
Czech	3	3
English	3	4
Finnish	6	6
French	4	4
German	3	3
Hindi	7	7
Icelandic	3	3
Indonesian	3	7

Language	PUD	PSUD
Italian	4	3
Japanese	4	7
Korean	7	7
Polish	3	3
Portuguese	3	3
Russian	3	3
Spanish	4	4
Swedish	3	3
Thai	6	6
Turkish	7	7

Table 8: Most voted best model for the distribution of syntactic dependency distances in sentences of fixed lengths, for every language and collection. The most voted best model is computed aggregating models by type, thus counting together the occurrences in which Models 3-4 (Models 6-7) are the best. Models 3-4 are marked with pink, Model 5 with yellow, and Models 6-7 with blue to ease visualization.

Language	PUD	PSUD
Arabic	5	5
Chinese	5	5
Czech	3-4	3-4
English	3-4	3-4
Finnish	6-7	6-7
French	3-4	3-4
German	3-4	3-4
Hindi	6-7	6-7
Icelandic	3-4	3-4
Indonesian	3-4	5

Language	PUD	PSUD
Italian	3-4	3-4
Japanese	3-4	6-7
Korean	6-7	6-7
Polish	3-4	5
Portuguese	3-4	3-4
Russian	3-4	5
Spanish	3-4	3-4
Swedish	3-4	3-4
Thai	5	5
Turkish	6-7	6-7

The best model for sentences of fixed lengths shows some variability for short and long sentences (Figure 6). Nevertheless, a double regime model is the most frequent best one across sentence lengths in 17/20 languages in PUD (including a tie between Model 5 and Models 6-7 in Chinese), and in 14/20 languages in PSUD (Table 8). Within the languages for which a two-regime model is the best one, Models 3-4 win in 13/17 languages in PUD, and in 9/14 in PSUD. Once again, we find high consistency between annotation styles, with the exceptions of Indonesian, Polish, and Russian, for which PSUD yields Model 5 as the most frequent best one (while PUD yields models 3-4), and Japanese, for which PUD and PSUD differ in the type of two-regime model. Finally, Model 5 is the most frequent best one in both collections for Arabic, Chinese, and Thai. However, Figure 7 shows how the most voted best model ceases to be Model 5 in some instances of both PUD and PSUD when the representativeness of a sentence length is taken into account. The only languages in which Model 5 is consistently the most frequent best one even after imposing an arbitrary high threshold are Thai, Indonesian, and Arabic in PSUD. Arabic shows a border-line behaviour in PUD, with Model 5 being consistently the most voted only up to a certain threshold value. Finally, a comparison of the actual distribution against the best model in sentences of fixed characteristic length is shown in Appendix E.

5.2 The break-point

When looking at languages globally, meaning considering jointly sentences of any length, we find that the break-point d^* always takes small values – ranging between 2 and 7 – and has a quite small standard deviation (Figure 8 and Table 9), meaning that its value is similar across languages. This is especially true for Models 3-4 and the PSUD collection: out of 11 languages having Models 3-4 as the best ones in this collection, 9 have an estimated break-point at $d^* = 4$ (Figure 8). In PUD these models have an average d^* value of 5, but with some more variability across languages. In both types of two regime models median and mean values are virtually the same, independently of annotation style, providing additional evidence for the low variance of d^* (Table 9). Checking the distribution of d^* within a language allows us to verify whether global values (found when mixing sentence lengths), namely the bars in Figure 8, are good approximations of the break-points actually observed in real sentences of any fixed length. We display the distribution of d^* across sentence lengths for each language in the same figure as a violin plot. Once again the median is very close to the mean in almost every combination of two-regime model and annotation style – with the exception of Models 6-7 in PSUD – further supporting H_2 (Table 10). Then, notice that where Models 3-4 are the best we observe relatively narrow distributions, skewed towards low values and showing one or a few modes (Figure 8). In particular, the global value of d^* is virtually

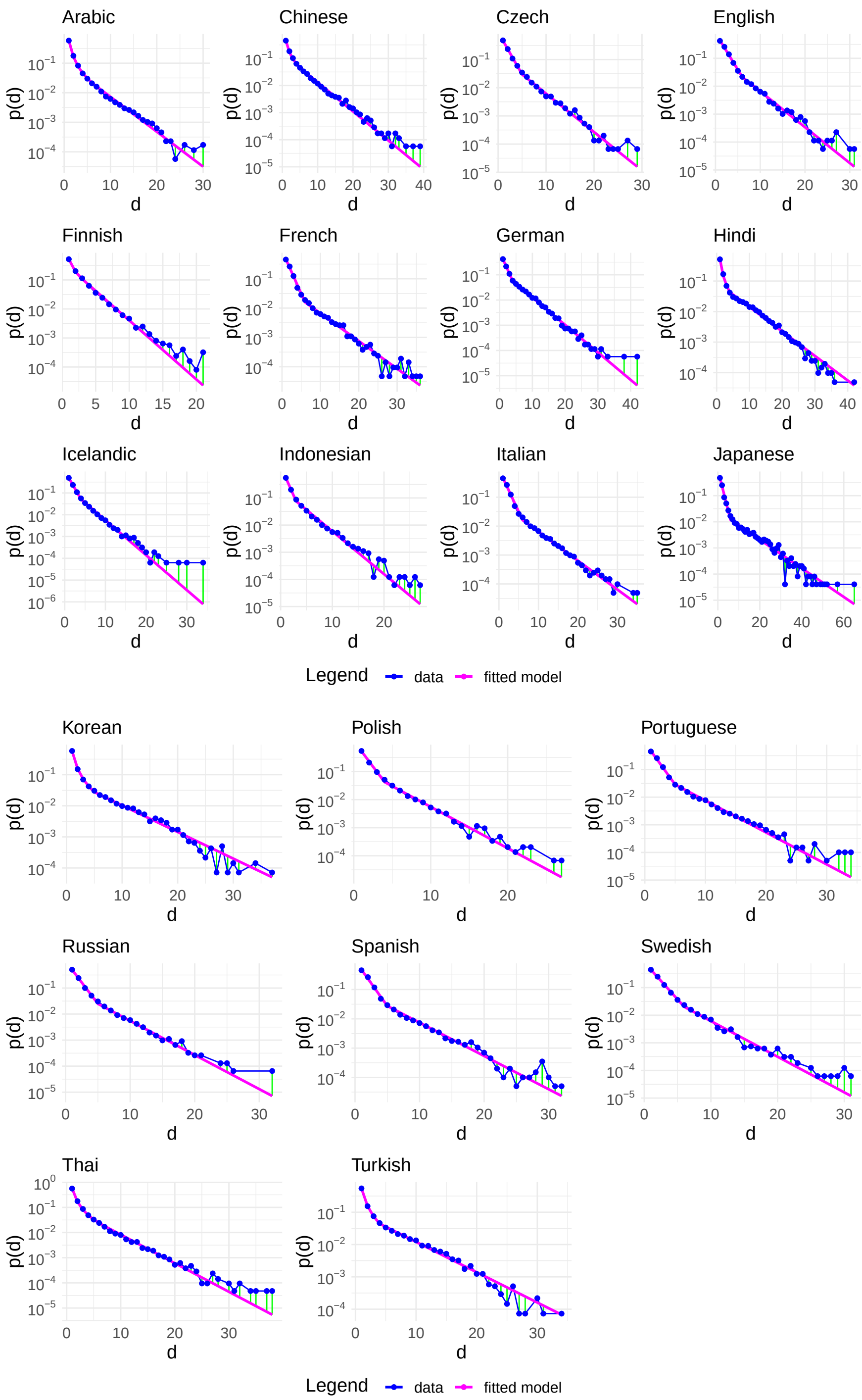
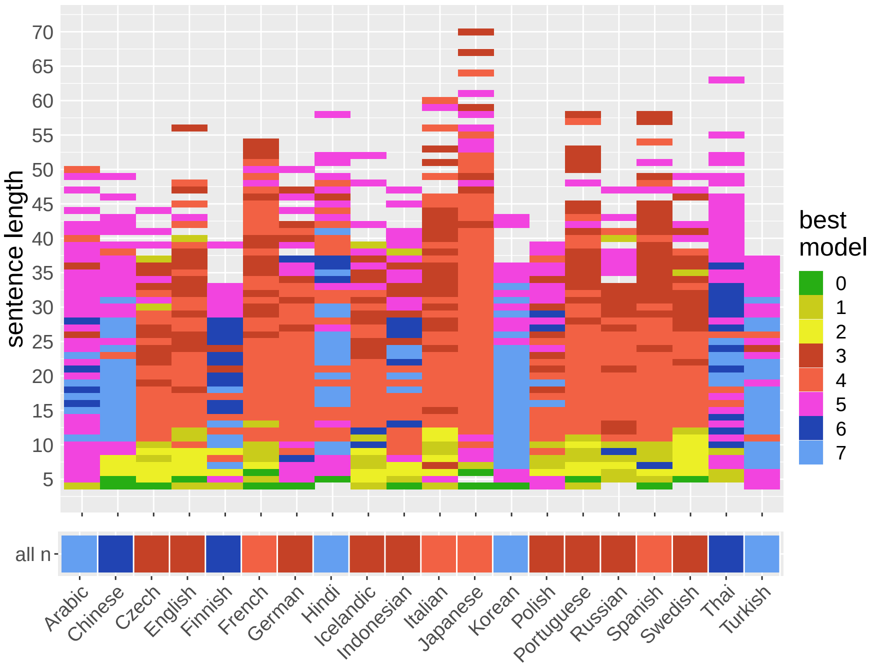
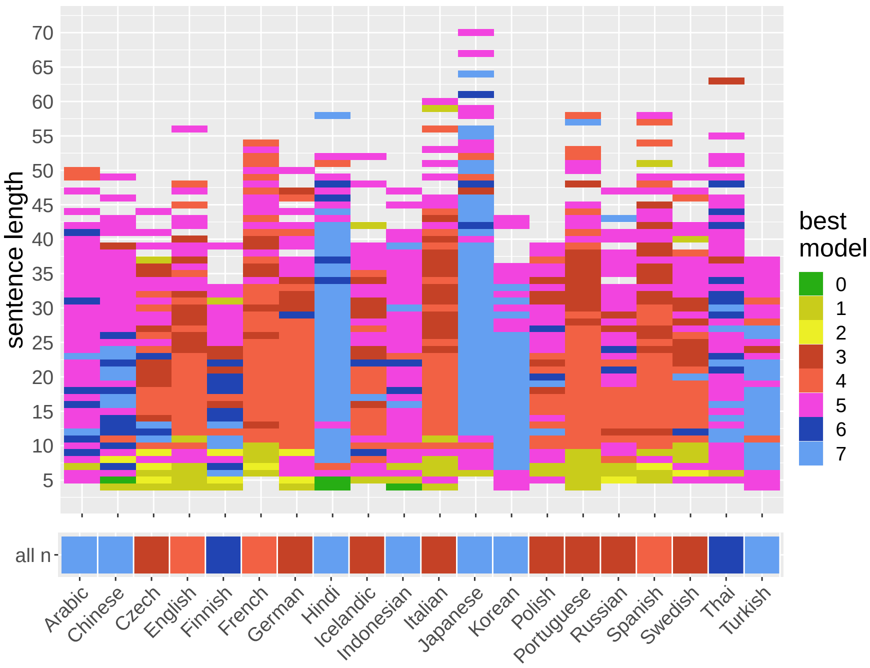


Figure 5: $p(d)$, the probability that a dependency link is formed between words at distance d according to the data and the best model for every language in PUD.



(a) PUD collection



(b) PSUD collection

Figure 6: Distribution of best model for each sentence length on top, with reference to the best model on mixed sentence lengths at the bottom. (a) PUD collection. (b) PSUD collection. In both (a) and (b) the empty tiles mark lengths for which no sentence was observed, or on which model selection was not performed given the minimum requirement to fit a double-regime model, described in section 4.1.3. Here Model 0 refers to Model 0.0.

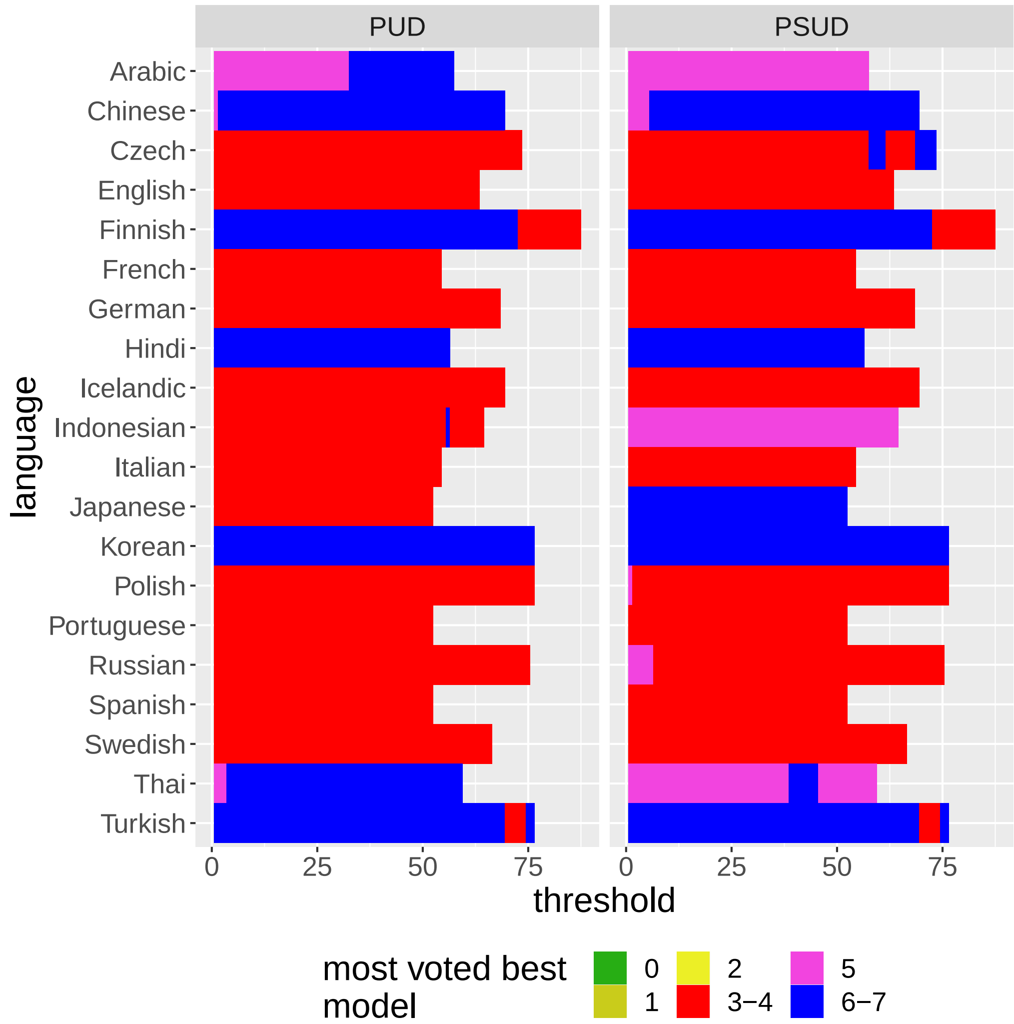


Figure 7: Most voted best model type across sentence lengths for increasing representativeness threshold. When no threshold is set (1 minimum sentence), we get the scenario displayed in Figure 6. Ties are counted in favour of models without two regimes. Here Model 0 refers to Model 0.0.

always found in correspondence of one of these modal values, confirming its representativeness for the whole language. Considering that sentences can reach up to a minimum of 37 (Turkish) and a maximum of 70 words (Japanese) (Table 3) the observed variation ranges in Models 3-4 are quite small, with values going up to roughly $d^* = 13$. On the other hand, within languages for which Models 6-7 are the best when mixing sentence lengths, the distribution of d^* across different sentence lengths is generally flatter, especially in PSUD. Even where values are centered around a mode, this does not correspond with the break-point estimated globally, with the exception of Hindi. Thus, it appears like the global break-points estimated in Models 3-4 are good approximations of the values observed within the language, while estimates of d^* in Models 6-7 are less reliable as representations of the actual break-point if there is any.

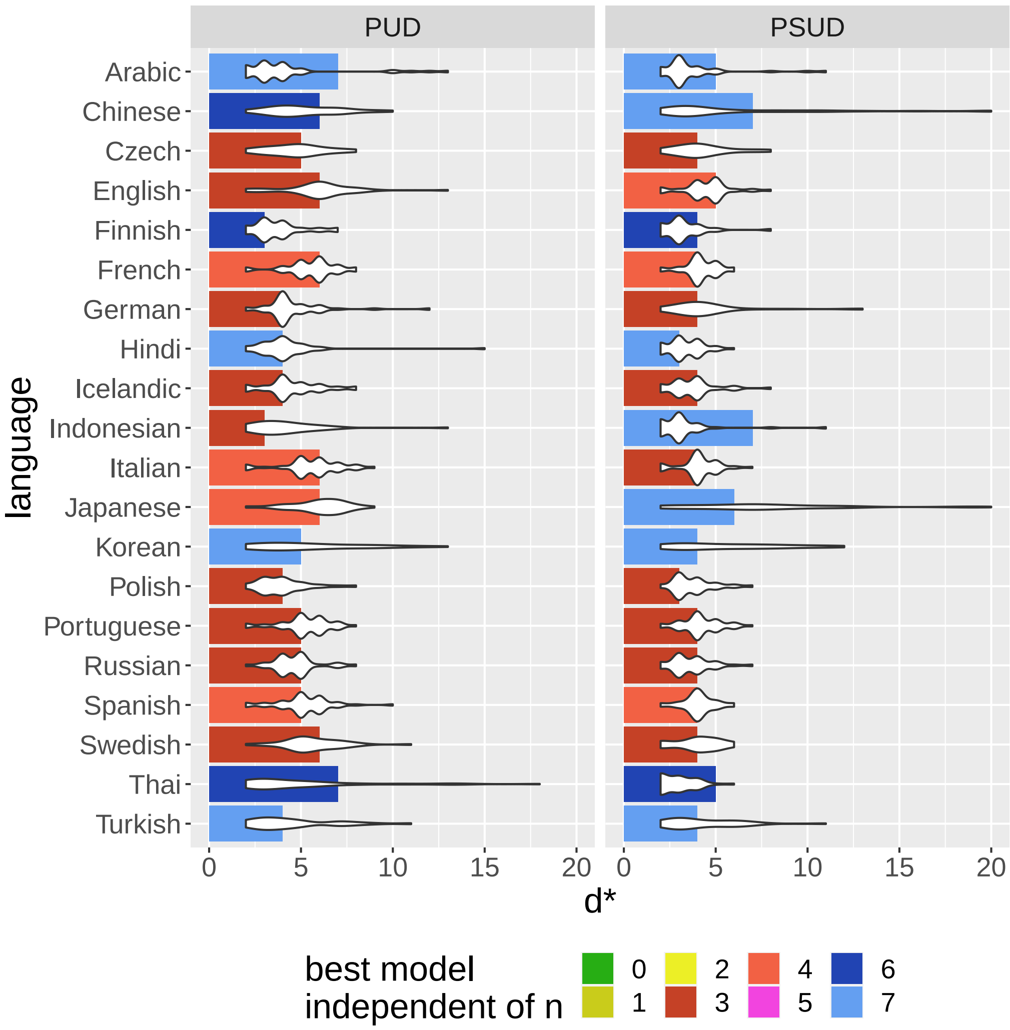


Figure 8: Value of d^* for mixed sentence lengths (bars) in each language and collection, and its distribution across fixed sentence lengths (violin plots), color-coded by best model independent of sentence length (namely the best model estimated on sentences of mixed lengths). Model 0 refers to Model 0.1 in the context of mixed sentence lengths.

Table 9: Summary statistics of the d^* parameter, by annotation style and type of two-regime model, estimated from model selection on sentences of mixed lengths. The summary is computed over languages where Models 3-4 are the best, where Models 6-7 are the best, and over all languages where a double-regime model is the best (Models 3-4-6-7). Thus, sample size is measured in number of languages. s stands for sample size, sd stands for standard deviation.

	Models	s	**Min.**	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max.**	sd
PUD	3-4	13	3.00	4.00	5.00	5.00	6.00	6.00	1.00
	6-7	7	3.00	4.00	5.00	5.14	6.50	7.00	1.57
	3-4-6-7	20	3.00	4.00	5.00	5.05	6.00	7.00	1.19
PSUD	3-4	11	3.00	4.00	4.00	4.00	4.00	5.00	0.45
	6-7	9	3.00	4.00	5.00	5.00	6.00	7.00	1.41
	3-4-6-7	20	3.00	4.00	4.00	4.45	5.00	7.00	1.10

Table 10: Summary statistics of d^* parameter, by collection and type of two-regime model, estimated from model selection on sentences of fixed lengths. The summary is computed over sentence lengths and languages where Models 3-4 are the best, where Models 6-7 are the best, and over all languages and sentence lengths where a double-regime model is the best (Models 3-4-6-7). Thus, sample size is measured in number of distinct sentence lengths. s stands for sample size, sd stands for standard deviation.

	Models	s	**Min.**	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max.**	sd
PUD	3-4	431	2.00	4.00	5.00	5.37	6.00	13.00	1.43
	6-7	134	2.00	4.00	6.00	6.28	7.00	18.00	3.03
	3-4-6-7	565	2.00	4.00	5.00	5.59	6.00	18.00	1.97
PSUD	3-4	297	3.00	4.00	4.00	4.32	5.00	13.00	1.11
	6-7	190	2.00	3.00	5.00	6.21	8.00	20.00	3.73
	3-4-6-7	487	2.00	4.00	4.00	5.06	5.00	20.00	2.65

5.3 Speed of decay

Recall that q_1 and q_2 are the slope parameters of Models 3-4, which quantify the speed of probability decay. For each language in which a two-regime model is the best, we consider q_1, q_2, and their ratio q_1/q_2, where the latter quantity is computed to establish which slope is steeper. It has been suggested that the probability decay is slower in the 2nd regime (Ferrer-i-Cancho, 2017; Ferrer-i-Cancho, 2004). When Models 6-7 are the best models, we estimate q_1 of the first regime by fitting the corresponding double exponential model (Model 3 or 4). When we refer to a slope, we refer to its absolute value.

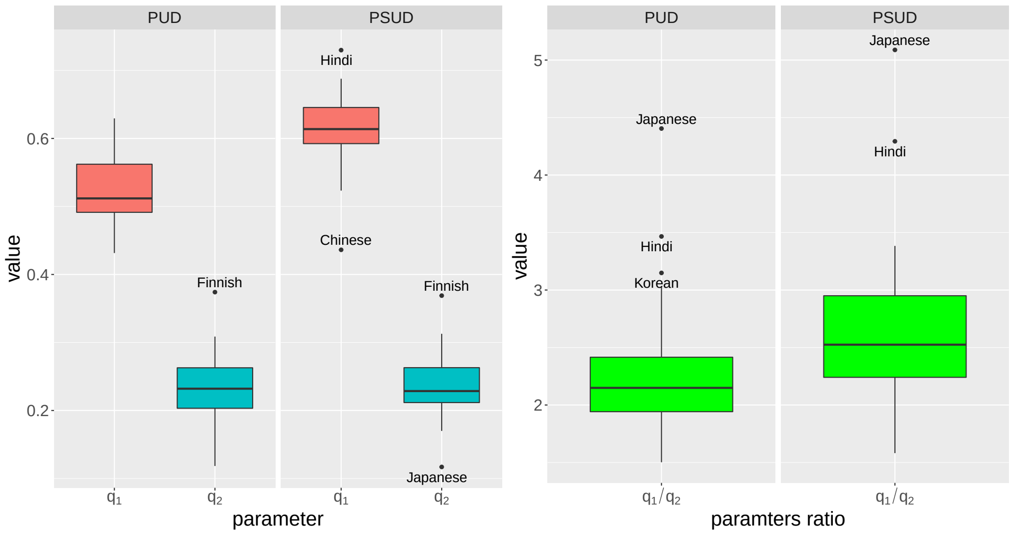
Where the best model has two regimes, the estimated slope parameters for each regime are fairly similar across languages (Figure 9 and Table 11). In addition, notice that the ratio q_1/q_2 is larger than 1 for every language and annotation style, and that q_1 and q_2 have a quite small standard deviation (Table 11). Standard deviation values are practically the same for the two parameters, but q_2 takes much lower values, meaning that it is relatively more variable than q_1. Moreover – as in the case of the break-point parameter – median and mean values are virtually the same, for both q_1 and q_2 and independently of annotation style. The slope estimated in the first regime in PUD is significantly lower than the one estimated in PSUD (Figure 9 (a)). Moreover, the estimated slopes show a clear pattern, with probability in the first regime consistently decaying faster compared to the second one. This pattern holds for the overwhelming majority of sentence lengths within a language, with a few exceptions found for very short sentences (Figure 10).

Table 11: Summary statistics of q_1 and q_2 parameters and their ratio (q_1/q_2) for model selection on sentences of mixed lengths, by annotation style (referred to as collection). Statistics are computed over all sentence lengths and languages for which a double-regime model is the best. sd stands for standard deviation.

	Collection	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
q_1	PUD	0.43	0.49	0.51	0.52	0.56	0.63	0.05
	PSUD	0.44	0.59	0.61	0.61	0.65	0.73	0.06
q_2	PUD	0.12	0.20	0.23	0.24	0.26	0.37	0.06
	PSUD	0.12	0.21	0.23	0.24	0.26	0.37	0.05
q_1/q_2	PUD	1.50	1.94	2.15	2.32	2.42	4.40	0.70
	PSUD	1.58	2.24	2.52	2.75	2.95	5.09	0.79

5.4 The best model versus the optimality of syntactic dependency distances

Ω is a new closeness score for syntactic dependency distances. The higher its value, the closer the syntactically related words. Refer to section 4.2 for further details on its properties and computation.



(a) q_1 and q_2 parameters distribution (b) Ratio $\frac{q_1}{q_2}$ distribution

Figure 9: Distribution of slope parameters q_1 and q_2 and their ratio. Isolated points are labelled with the corresponding language.

The score takes positive values when syntactic dependency distances are minimized, negative values when they go against minimization, and values around 0 when there is no pressure in either direction (Ferrer-i-Cancho et al., 2022). Let $\langle \Omega \rangle$ be the average value of Ω over all sentences with a given length in a language. See Figure 11 and Figure 12 for the best model for each sentence length (a) and the corresponding value of $\langle \Omega \rangle$ (b), for PUD and PSUD respectively. First, in sentences of a very few words, the best model is either Model 0 or one with a single regime, and the values of the optimality score signal the coexistence of the three possible systems: anti-DDm (orange tiles), no bias (white tiles), and pro-DDm (purple tiles). Given the definition of the score, we expect that, under the assumption that Model 0 is the real distribution, $\langle \Omega \rangle$ will take values around 0, as both situations underlie random word ordering. This expectation is met in 6/8 instances, as displayed in Table 12, and as suggested by the correspondence between white tiles in (b) and green tiles in (a). The two exceptions are Korean in PSUD and Polish in PUD, for which the best model is Model 5. Then, for sentences longer than 5-6 words, $\langle \Omega \rangle$ indicates that distances in syntactic structures are always minimized, which is mirrored in the disappearance of Model 0 and the predominance of the single regime models. Finally, as pressure for minimization further increases with sentence length, these simpler models are progressively replaced by the models with two regimes.

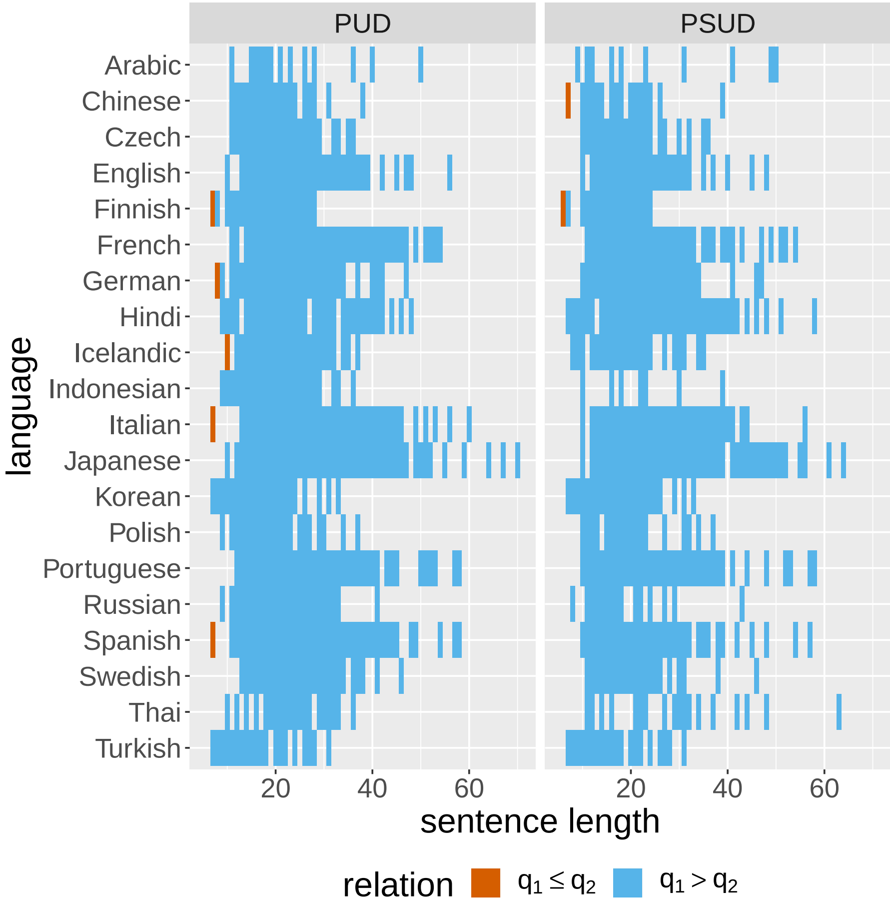
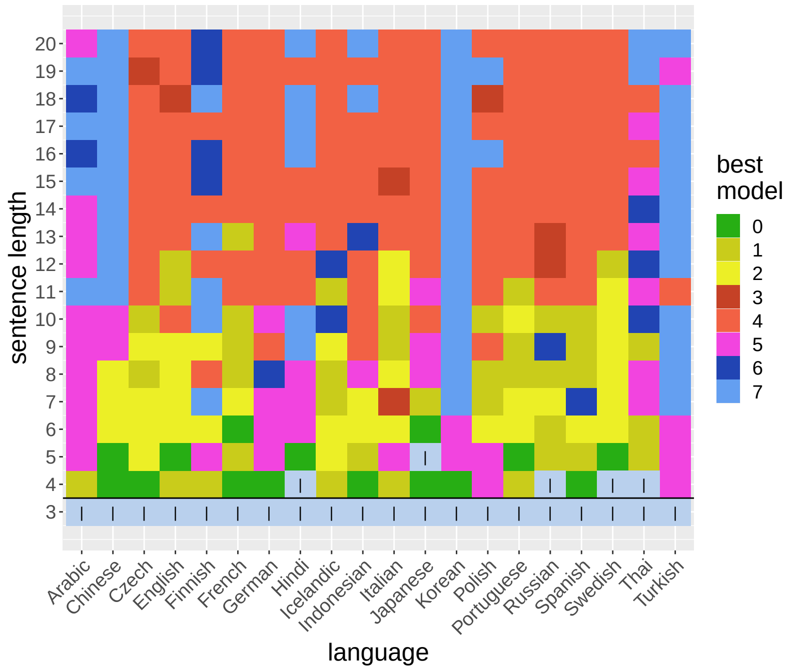


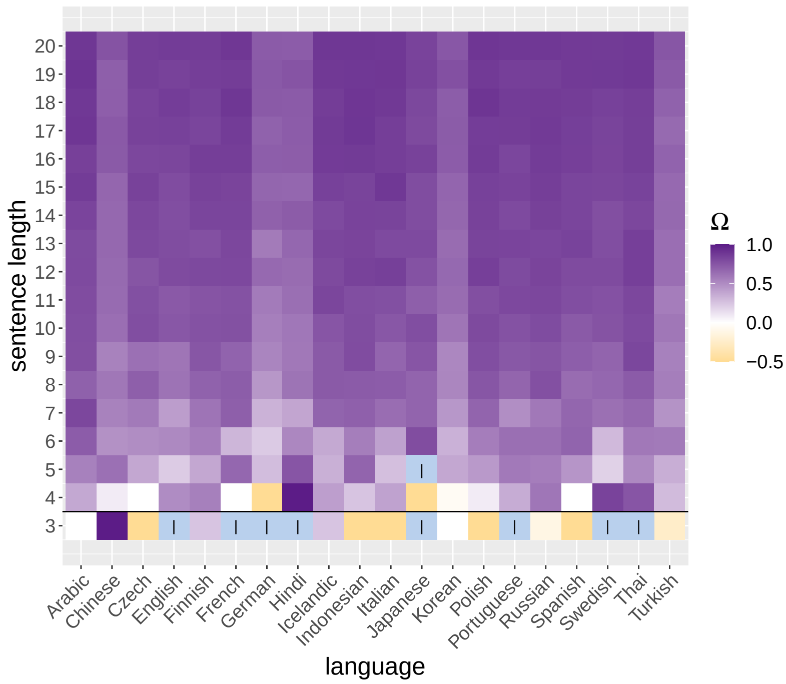
Figure 10: Relation between slope parameters q_1 and q_2 estimated from model selection on fixed sentence lengths. Lengths for which $q_1 \leq q_2$ are colored in red, while those for which $q_1 > q_2$ are colored in blue. Where the best model was 6 (7), the first slope was approximated by fitting Model 3 (4) with the original value of d^*. The empty tiles indicate lengths for which no sentence was observed, a two-regime model was not the best one, or on which model selection was not performed given the minimum requirement on the number of observed distance values to fit a double-regime model, described in section 4.1.3.

Table 12: Estimated best model on fixed sentence in collections, languages, and sentence lengths for which $|\langle\Omega\rangle - \epsilon| \leq 0$, with $\epsilon = 0.1$. $\langle\Omega\rangle$ is the average value of Ω over all sentences with a given length in a language.

Collection	Language	n	$\langle\Omega\rangle$	Best model
PUD	Korean	4	−0.05	0
	Czech	4	0.00	0
	French	4	0.00	0
	Spanish	4	0.00	0
	Polish	4	0.08	5
	Chinese	4	0.08	0
PSUD	Korean	4	−0.10	5
	Hindi	4	0.00	0

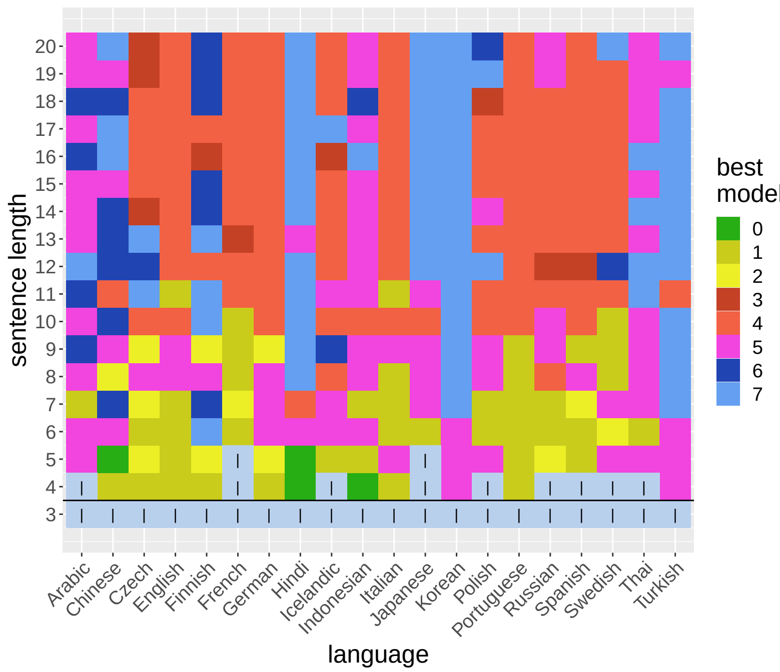


(a) Best model for every language and sentence length.

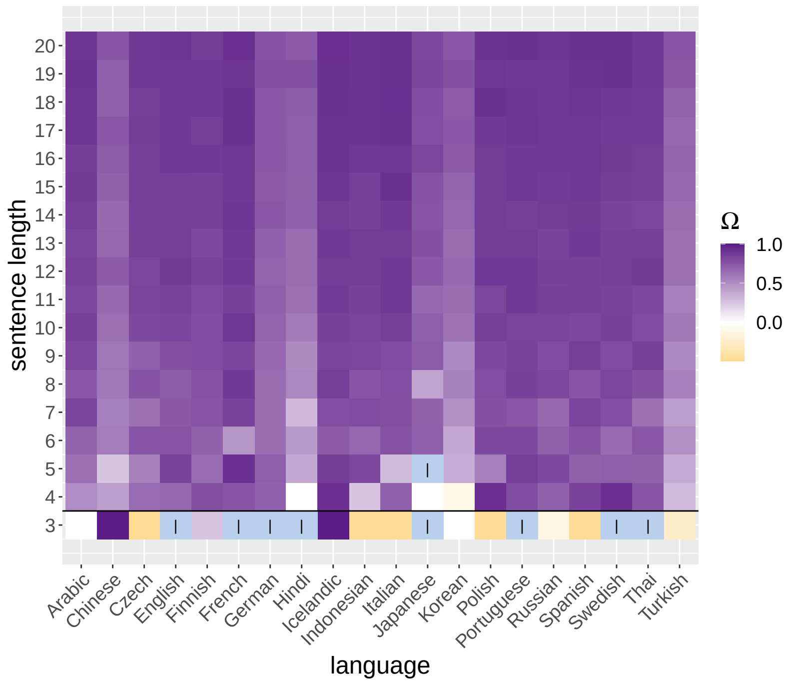


(b) ⟨Ω⟩ for each language and sentence length: orange signals negative values, white signals values around 0, and purple signals positive values.

Figure 11: Relation between Ω score and best model in PUD. The barred gray cells indicate the sentence lengths which have not been observed, or that were excluded from model selection according to the representativeness threshold. Sentence lengths are cut at $n = 20$ to ease visualization. Model 0 refers to Model 0.0. In (b), orange signals negative values, white signals values around 0, and purple signals positive values.



(a) Best model for every language and sentence length.



(b) $\langle \Omega \rangle$ for each language and sentence length: orange signals negative values, white signals values around 0, and purple signals positive values.

Figure 12: Relation between Ω score and best model in PSUD. The format is the same as in Figure 11.

6 Discussion

First, we focus on the two hypotheses object of study, namely that syntactic dependency distances are distributed following two exponential regimes (H_1), and that the break-point shows low variation across languages (H_2). Our results provide strong evidence for both hypotheses in a large group of languages, mainly Indo-European, consistently across annotation styles. Second, we reflect on the parameters yielding the best fit and pay attention to the greater steepness of the first regime with respect to the second one, and the homogeneity of the estimated slopes across languages. Finally, we discuss the relation between the best estimated model and the closeness of syntactic dependencies as captured by the optimality score Ω (Ferrer-i-Cancho et al., 2022), and summarize the effect of annotation style.

6.1 The reality of two regimes

6.1.1 The shape of the distribution

As it is often the case, the path to the truth seems to lie in the middle. We could neither generalize to all languages hypothesis H_1 (supported by 13/20 languages in PUD and by 11/20 in PSUD), first advanced by Ferrer-i-Cancho (2004), nor fully corroborate the finding that dependency distances are power-law distributed as reported for Chinese (Liu, 2007). However, we provided evidence for a possible explanation integrating both: a two-regime model in which the first regime is either exponential or power-law distributed, and the second one follows an exponential decay. A two-regime model is found in all languages when mixing sentences of different lengths (Table 7), while two regimes are robustly found for the majority of languages when specific sentence lengths are considered (Figure 6). However, while the picture is clear and consistent in the first case, discussion on sentences of specific lengths requires further elaboration. The shape of the distribution depends on the length of the sequence (Figure 6), which is expected by the relation between DDm and sentence length. Processing short distances implies lower cognitive effort and robust statistical evidence suggests that DDm might irrelevant or be canceled out by other word order principles in short sequences (Ferrer-i-Cancho, 2024; Ferrer-i-Cancho and Gómez-Rodríguez, 2021; Ferrer-i-Cancho et al., 2022). Then, the varying intensity of the pressure for minimization yields different distributions in different areas of the sentence length domain, which we characterized with the following (potentially overlapping) regions (Figure 6, Figure 11 and Figure 12).

- *Random linear arrangement.* In short sentences (approximately $n \leq 6$) DDm might be neglectable or weak enough to be surpassed by other word order principles (Ferrer-i-Cancho, 2024; Ferrer-i-Cancho and Gómez-Rodríguez, 2021; Ferrer-i-Cancho et al., 2022), resulting in Model 0 (green tiles in Figure 6, Figure 11 and Figure 12) sometimes being the best one to describe the distribution. Where it is not Model 0, a model with a single regime is the best one.

- *Single chunk.* Up to roughly 13 words the best model is mainly one of 1, 2, or 5 (yellow and pink tiles in Figure 6, Figure 11 and Figure 12) in most languages. This possibly indicates that the sentence can be processed as a single chunk when the number of words is small enough, and dependencies must be highly local to allow for this.

- *Two regimes.* The bulk of the sentence length domain is characterized by the presence of two-regime models (red and blue tiles in Figure 6, Figure 11 and Figure 12). In these longer sentences the burden on STM becomes heavier, and two regimes might emerge from the breaking down of the sentence into chunks. After a very steep decrease in probability, a long dependency becomes more likely in order to link a chunk to the previous one.

- *No consistent pattern.* For long (and rare) sentences no clear pattern appears, as the scarcity of examples for large sentence lengths introduces variability in the estimation of the best model.

6.1.2 On power laws

When mixing sentences of distinct length, the best model is always a two regime model (Table 7). Across sentence lengths, the majority of languages have a model with two regimes as the most frequent best one, and a few languages in both collections show a power-law behavior (Table 8). Nevertheless, setting a rather high representativeness threshold dramatically reduces evidence for single-regime power-law, especially in PUD (Figure 7). This is for instance the case with Chinese in both collections. In spite of this, for Arabic, Indonesian, and Thai the most frequent best model is robustly Model 5 when the SUD annotation style is used.

Although Chinese has been argued to follow a single-regime power law (Liu, 2007), our findings indicate that Chinese is better fitted by a two-regime model with an initial power-law regime (Model 6 or 7) when mixing sentences of any length (Table 7). However, if the representativeness threshold is set to a low value (Figure 7), a single-regime power law (Model 5) can be retrieved, but such a low threshold casts doubts on the statistical strength of the best model when mixing sentences of distinct length. In contrast, the claim of a power law for Chinese is supported clearly for sentences of fixed length, where Model 5 is the most frequent best model across sentence lengths (Table 8).

Overall, two exponential regimes are the most common distribution for both mixed and fixed sentence lengths. However, what our analysis also proposes is that power laws can well describe the distribution in the first regime for some languages (mainly non Indo-European) when sentence lengths are mixed, as well as the distribution for specific sentence lengths for a small subset of them. Importantly, power-laws can also arise from undersampling, as highlighted by our representativeness analysis (Figure 7). In previous research it has been argued that power-laws could emerge from mixing sentence lengths in which distances are distributed following an exponential curve (Ferrer-i-Cancho and Liu, 2014; Stumpf and Porter, 2012). Our research invalidates this argument (at least in the scope of our sample of languages), and identifies instances of another sort of mixing: for Arabic, Indonesian, and Thai in PSUD, mixing sentence lengths that are individually power-law distributed results in a distribution with two regimes with a power-law in the 1st regime, suggesting that further investigation is required in this direction.

6.1.3 Tail variability

Plots of the best model against the real data allows one to visually assess the quality of its fit to the data (Figure 5 for PUD and Figure 15 for PSUD). The best models are able to very well capture the shape of the bulk of the distribution and the initial bending in all languages. However, they are not always able to fully capture the variability along the tail of the distribution. To begin with, noise naturally emerges for longer distances, which belong to rare long sentences. As we explained above, there are lengths for which only one sentence is observed. Taking this into account, the deviation from the best model could suggest the possible presence of an unveiled pattern for some languages. We hypothesize the existence of more than one break-point, implying incremental executions of a “chunk-and-pass” mechanism (Christiansen and Chater, 2016).

However, introducing more regimes would greatly increase both the complexity of estimation (maximum likelihood estimation already requires putting particular care in the estimation of 3/4 parameters, see section 4), and the risk of overfitting the data. Thus, a thorough and rigorous methodology would need to be employed for such modelling, which should be the subject of future research.

6.2 The homogeneity of the break-point

The break-point values we estimated are largely homogeneous across languages, and average values of 5 (PUD) and 4 (PSUD) words, with small variation. These values are consistent with the literature on limitations of short term memory: in no language d^* exceeds the “magical number” 7 (Miller, 1956), and the bulk of the values is centered at 4 ± 1, which is generally recognized to be the working memory limitation on a wide range of tasks (Cowan, 2001).

Nevertheless, some variability can still be observed, especially among the break-points of sentences of different lengths within a language. In fact, an implicit assumption of H_2 is that the value estimated globally for a given language is a reliable approximation of the constraint acting at the sentence level, and this can be verified by looking at the break-points estimated for each given length. We find that for languages in which H_1 holds (two exponential regimes), the distribution of d^* across sentence lengths is very narrow, and centered around the global value of d^*. The break-points estimated in Models 6-7 are more variable, but they still vary in a rather small range compared to the range of variation of the actual sentences (Table 3 and Table 4).

The average length in words of simple declarative sentences is 3.7 (from 2.6 in Turkish up to 5.4 in Mandarin) (Fenk-Oczlon and Pilz, 2021).[1]. We believe that this variability in the size clauses is captured by our breakpoint (Figure 8) but this issue should be the subject of future research with a linguistic or cognitive focus.

6.3 Patterns in probability decay across regimes

Given the large applicability of the two-regime models, we take closer look to the speed of probability decay. The slopes observed across languages are quite narrowly distributed around the same values (Figure 9). It is interesting to notice that while the first slope is significantly larger in PSUD, q_2 shows little variation in the two collections. This suggests that, depending on annotation style, the distribution of the dependencies within word chunks will change, but beyond word chunks, the chunking mechanism follows a similar structure. Another interesting pattern concerns the steepness of the first regime with respect to the second one. When mixing sentences of different lengths the first regime is always steeper than the second one (Figure 9) and this is virtually always the case even when considering specific sentence lengths, with a very few exceptions in short sentences (Figure 10). This provides additional support for the "chunk-and-pass" paradigm (Christiansen and Chater, 2016). An explanation for that pattern could be that, when memory limits are approached in long enough sentences, the current chunk needs to be closed, and a new longer dependency becomes more likely in order to link the forthcoming chunk (thus reducing the speed of probability decay). The two regimes (and in particular Model 4) may be found even if the real distribution is Model 0, given their similar BIC scores (Figure 13). However, Model 4 could only mimic a linear curve (Model 0) if the second regime was steeper than the first one.

6.4 The best model versus the optimality of syntactic dependency distances

In section 6.1, we have described how the shape of the distribution varies depending on sentence length. Here, we aim to understand the interplay with different degrees of pressure for DDm for long versus short sentences. Previous research has pointed out at how $\langle\Omega\rangle$ is smaller in short sentences, likely due to DDm being neglected or canceled out by other word order principles (Ferrer-i-Cancho, 2024; Ferrer-i-Cancho and Gómez-Rodríguez, 2021; Ferrer-i-Cancho et al., 2022). We provide additional evidence for this phenomenon by unravelling a direct correspondence between sentences where $\langle\Omega\rangle$ is close to 0, and those in which the best model is Model 0 (Table 12). Moreover, we observe a relation between the intensity of DDm and the best model for the distribution. Namely, as pressure for minimization increases with sentence length, the best model changes (Figure 11 and Figure 12). While correlation does not

[1] The data can be found in the Supplementary Material (Sheet 1)

imply causation, it is crucial to understand that both the pressure for DDm and the best model for the distribution of syntactic dependency distances are not homogeneous through sentence length. Thus, distances belonging to sentences of different length are subject to different pressures, and this should be taken into account when trying to model the distribution. In particular, these different levels of pressure could yield different mechanisms. Indeed, the more complex distributions – those with two regimes – tend to emerge for long enough sentence length, when the pressure for DDm is stronger, likely calling for a structured processing mechanism.

6.5 The effect of annotation style

So far we have observed commonalities and differences between PUD and PSUD. Overall, the main qualitative results are robust to annotation style, supporting the soundness of the observed patterns, but some differences emerge. The discussion on the origins of such differences is open, and is connected to the fundamental question of whether an annotation style is a more accurate representation of our brain's functioning or the linguistic processing than the other, or whether different styles simply mirror different aspects of this functioning or processing. While providing a rather descriptive account of such differences, we partly attempt to address this question.

6.5.1 The shape of the distribution

The first main point concerns the very high consistency in the best estimated models (Figure 8). However, there are a few exceptions, which we classified in two types: differences in right truncation, and in the distribution in the first regime. The latter is clearly of greater interest and it concerns two languages, Japanese and Indonesian, both having Models 3-4 as the best model in PUD, and Model 7 in PSUD, but showing a very different behaviour. For Japanese, the best models estimated on specific sentence lengths and by mixing all sentence lengths are highly consistent within each collection, and in both cases the break-point value is $d^* = 6$. This suggests a real difference in probability decay within a chunk depending on the chosen annotation guidelines, but also conveys the concreteness of the quantified limit on memory for such language. On the other hand, for Indonesian we find mixed evidence, both in terms of estimated break-point, which goes from $d^* = 3$ in PUD to $d^* = 7$ in PSUD, and in terms of best model for fixed sentence lengths (which is consistently a one-regime power-law in PSUD). In fact, this takes us to one of the main differences between annotation styles (Figure 7): while in PUD the only language showing some evidence for a single power-law regime for fixed sentence lengths is Arabic, in PSUD we have three languages strongly supporting the reality of such distribution. For Arabic, Indonesian, and Thai, the two regimes observed for mixed sentence lengths contradict what is found when sentence lengths are analysed in isolation. This seems to reflect Simpson's paradox, a phenomenon according to which a statistical trend disappears when single groups are considered, and suggests that there is some variability left to explain.

6.5.2 The break-point

We have seen in Figure 8 how the break-points estimated in both collections cover the same portion of domain, ranging from 3 to 7. However, while in PUD there is no settling around a particular value, in PSUD d^* is nearly uniform at $d^* = 4$, especially within Models 3-4. This raises the following questions: is this regularity given by chance? Or does it mirror a better ability of SUD to capture syntactic relations as formed by our minds? Given that – besides individual differences – the overall structure of the brain is assumed to be the same for all humans, the constraint on memory is expected to be uniform across languages (hence the motivation for H_2). Thus, one could speculate that SUD annotation style is actually more capable of unveiling this uniformity, that is assumed to exist.

6.5.3 Dependency distance minimization

SUD guidelines have been found to lead to shorter dependency distances (Ferrer-i-Cancho et al., 2022; Osborne and Gerdes, 2019; Yan and Liu, 2021). When dependency distances are conveniently normalized with respect to the gap between the random baseline and the minimum baseline, SUD reflects distances that are closer to optimality (Ferrer-i-Cancho et al., 2022). Such ability of SUD to reflect dependency distance minimization of effects is confirmed by our findings. In fact, despite predicting a power-law decay in the first regime for two more languages compared to PUD, q_1 is significantly higher in PSUD (Figure 9). This entails a faster decay in probability within the chunk, related to the predominance of short local dependencies in PSUD. Moreover, the values of Ω computed in the PSUD collection are generally larger (tiles in Figure 12 (b) are darker than in Figure 11 (b)), confirming a stronger degree of optimization of dependency distances in the SUD framework (Ferrer-i-Cancho et al., 2022).

7 Conclusion

Two decades after the first observations on the peculiar shape of the distribution of syntactic dependency distances (Ferrer-i-Cancho, 2004), some new light has been shed. A crucial finding is that the probability of observing a dependency – independently of the length of the sentence it belongs to – is best described by a double-regime model. Furthermore, the finding also holds at a finer-grained level, distinctively considering each sentence length. In this setting, for the great majority of languages a double-regime model is the most frequent one, while the few remaining languages show a power-law decay as the most frequent, partly in accordance with what has been found concerning a Chinese treebank, where however sentences of mixed lengths were analysed (Liu, 2007). Furthermore, the break-point between the two regimes estimated globally for each language varies in a small range ($3 \leq d^* \leq 7$), which becomes even narrower when only languages in which H_1 holds are considered. In fact, H_2 seems to be related to the probability distribution observed in the first regime, leading to the identification of a group of languages where probability follows a two-regime exponential decay (H_1), and within which the break-point is very similar (H_2). This group is mainly populated by Indo-European languages. However, languages from this family are over-represented in our sample, and other interesting patterns could emerge if a larger group of languages from other families where analysed. These considerations hold independently of annotation style, but it has not escaped our attention that in PSUD values of d^* for such group are almost uniform at 4, a widely accepted quantification of the constraint on short term memory (Cowan, 2001). This could, in our opinion, reflect a higher sensitivity of SUD annotation style to the way in which our minds create and process language, bringing to light a "universal" constraint which is not language dependent. Another general pattern emerged is the relation between the speeds of the decays, whereas probability in the first regime is always faster than in the second one. As already pointed out, this result may look paradoxical: if cognitive pressure induces a decay in probability as syntactic dependency distance increases, why does such a decay slows down beyond the breakpoint? (Ferrer-i-Cancho, 2017)? In the framework of language processing, these findings provide strong support for the "chunk-and-pass" mechanism (Christiansen and Chater, 2016). In fact, the presence of these two different regimes could actually mirror the two different speeds at which probability decays within a chunk and beyond it. In physical terms, the true units of measurement of distance may change: within the word chunk the unit of distance are words whereas, beyond the word chunk, the actual distance may be chunks in the hidden space of incremental processing of the sentence. The breakpoint and the slow down after the breakpoint may arise because we have imposed the use of words as unit of measurement independently of the stage of syntactic parsing. In our view, this appears to be the most reasonable and pertinent explanation for the observed systematic decrease in the strength of DDm, but we do not exclude that other explanations could as well be plausible. Future work could further investigate the distribution in the second regime, exploring different combinations of exponential and power-law decay. Then, the possible presence of more than one break-point could be explored. Importantly, to understand the extent to which the observed phenomena can be considered universal, the same analysis shall be performed on a wider set of languages.

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Appendices

A Model derivation

Here we detail the mathematical derivation of the non-standard models in Section 2.

Model 0.1 We consider a general model for sentences of varying length, defined as

$$p(d) = \sum_{n=min(n)}^{max(n)} p(d|n)\, p(n),$$

where $p(d|n)$ is the conditional probability of d given that the sentence length has n words, $p(n)$ is the proportion of sentences having length n, and $\min(n)$ and $\max(n)$ are the minimum and maximum observed values of n in the sample. By definition, $p(d|n)$ satisfies two conditions, i.e. $p(d|n) = 0$ when $d \notin [1, n)$ and

$$\sum_{d=1}^{n-1} p(d|n) = 1.$$

Thanks to these two conditions, it is easy to see that $p(d)$ is properly normalized, that is

$$\begin{aligned} \sum_{d=1}^{max(n)-1} p(d) &= \sum_{d=1}^{max(n)-1} \sum_{n=min(n)}^{max(n)} p(d|n)\, p(n) \\ &= \sum_{n=min(n)}^{max(n)} p(n) \sum_{d=1}^{n-1} p(d|n) \\ &= 1 \end{aligned}$$

By setting $p(d|n)$ according to the null hypothesis of a random shuffling of the words of a sentence of n words (1), which satisfies the two conditions above, we obtain

$$p(d) = \sum_{n=min(n)}^{max(n)} \frac{n-d}{\binom{n}{2}}\, p(n).$$

Model 2 We define the cumulative distribution of Model 1 as

$$P_1(d) = \sum_{d'=1}^{d} p_1(d').$$

where $p_1(d)$ is defined as in 2. Model 2 is derived via renormalization of Model 1 after right-truncation, that is

$$p_2(d) = \frac{p_1(d)}{P_1(d_{max})},$$

where

$$\begin{aligned} P_1(d_{max}) &= \sum_{d=1}^{d_{max}} q(1-q)^{d-1} \\ &= 1-(1-q)^{d_{max}}. \end{aligned}$$

Hence

$$p_2(d) = \frac{q(1-q)^{d-1}}{1-(1-q)^{d_{max}}}.$$

Double-regime models Now we use $p_1(d)$ to refer to the definition of $p(d)$ for $d \leq d^*$ and $p_2(d)$ to refer to the definition of $p(d)$ for $d \geq d^*$. The definition of Models 3, 4, 6, 7 follows the template

$$p(d) = \begin{cases} p_1(d) = c_1 f_1(d) & \text{if } d \leq d^* \\ p_2(d) = c_2 f_2(d) & \text{if } d^* \leq d \leq d_{max}, \end{cases}$$

For models 3 and 6, one simply sets d_{max} to ∞. Thus, the assumption $p_1(d) = p_2(d)$ yields

$$c_2 = \tau c_1$$

with

$$\tau = \frac{f_1(d)}{f_2(d)}.$$

Recalling the definitions of the models (Table 1), it is easy to see that, for models 3 and 4,

$$\tau = \frac{(1-q_1)^{d^*-1}}{(1-q_2)^{d^*-1}}.$$

whereas for models 6 and 7,

$$\tau = \frac{d^{*^{-\gamma}}}{(1-q)^{d^*-1}}.$$

Let us derive the normalization factor c_1 for Models 3, 4, 6, 7 with the help of

$$\begin{aligned} S_1 &= \sum_{d=1}^{d^*} f_1(d) \\ S_2 &= \sum_{d=d^*}^{d_{max}} f_2(d). \end{aligned}$$

The normalization condition

$$\sum_{d=1}^{d_{max}} p(d) = 1$$

yields

$$c_1 = \frac{1}{S_1 + \tau S_2}. \tag{11}$$

For Models 3 and 4, S_1 is

$$S_1 = \sum_{d'=0}^{d^*-1} (1-q_1)^{d'} = \frac{1-(1-q_1)^{d^*}}{q_1}.$$

S_2 depends on the truncation point. For Model 3, the assumption $q > 0$ (thus $\lim_{d_{max}\to\infty}(1-q)^{d_{max}} = 0$) produces

$$\begin{aligned} S_2 &= \sum_{d'=d^*}^{\infty} (1-q_2)^{d'} \\ (1-q_2)S_2 &= S_2 - (1-q_2)^{d^*} + (1-q_2)^{\infty} \\ S_2 &= \frac{(1-q_2)^{d^*}}{q_2} \end{aligned} \tag{12}$$

By substituting S_1, S_2 and τ in 11, c_1 for Model 3 becomes

$$c_1 = \frac{q_1 q_2}{q_2 + (1-q_1)^{d^*-1}(q_1 - q_2)}$$

after some algebra.

In Model 4, probabilities are restricted up to d_{max}, thus

$$S_2 = \sum_{d'=d^*}^{d_{max}-1} (1-q_2)^{d'} = \frac{(1-q_2)^{d^*} - (1-q_2)^{d_{max}}}{q_2}. \tag{13}$$

Again, plugging S_1, S_2, and τ into 11 produces c_1 for Model 4, that is

$$c_1 = \frac{q_1 q_2}{q_2 + (1-q_1)^{d^*-1}(q_1 - q_2 - q_1(1-q_2)^{d_{max}-d^*+1})}$$

after some algebra.

For the second pair of double-regime models (Models 6 and 7), combining a zeta and a geometric distribution, S_1 is

$$S_1 = \sum_{d=1}^{d^*} d^{-\gamma} = H(d^*, \gamma),$$

while the second regime is shared with Models 3-4, so that S_2 corresponds to 12 for Model 6 and to 13 for Model 7. Then, the normalization factors are obtained again through 11, so that for Model 6

$$c_1 = \frac{q}{qH(d^*, \gamma) + d^{*-\gamma}(1-q)},$$

while for Model 7

$$c_1 = \frac{q}{qH(d^*, \gamma) + d^{*-\gamma}(1 - q - (1-q)^{d_{max}-d^*+1})}$$

after some algebra.

B Log-likelihood functions

In our setting, the log-likelihood of a model is

$$\mathcal{L} = \log \prod_{i=1}^{N} p(d_i) = \sum_{i=1}^{N} \log p(d_i) = \sum_{d=1}^{max(d)} f(d), \log p(d).$$

Next we derive the log-likelihood functions for each model with the help of Table 1.

For Model 0.0, where d_{max} is the only free parameter, we have

$$\begin{aligned}
\mathcal{L} &= \sum_{d=1}^{max(d)} f(d) \log \left(\frac{2(d_{max}+1-d)}{d_{max}(d_{max}+1)} \right) \\
&= \sum_{d=1}^{max(d)} f(d) \left[\log \left(\frac{2}{d_{max}(d_{max}+1)} \right) + \log(d_{max}+1-d) \right] \\
&= N \log \left(\frac{2}{d_{max}(d_{max}+1)} \right) + W,
\end{aligned}$$

where

$$\begin{aligned}
N &= \sum_{d=1}^{max(d)} f(d) \\
W &= \sum_{d=1}^{max(d)} f(d) \log(n-d).
\end{aligned}$$

For Model 0.1, in which the observed sentence lengths are supplied and there is no free parameter, we have

$$\begin{aligned}
\mathcal{L} &= \sum_{n=min(n)}^{max(n)} \sum_{d=1}^{max(d)} f(d) \log \frac{2(n-d)}{n(n-1)} \\
&= \sum_{n=min(n)}^{max(n)} \sum_{d=1}^{max(d)} f(d) \left[\log \frac{2}{n(n-1)} + \log(n-d) \right] \\
&= \sum_{n=min(n)}^{max(n)} \left[N_n \log \frac{2}{n(n-1)} + W_n \right]
\end{aligned}$$

where

$$\begin{aligned}
W_n &= \sum_{d=1}^{max(d)} f(d) \log(n-d) \\
N_n &= \sum_{d=1}^{max(d)} f(d)
\end{aligned}$$

in sentences of length n. For the *geometric models*, we start from the derivation of the right-truncated version, namely Model 2

$$
\begin{aligned}
\mathcal{L} &= \sum_{d=1}^{max(d)} f(d) \log \frac{q(1-q)^{d-1}}{1-(1-q)^{d_{max}}} \\
&= \sum_{d=1}^{max(d)} f(d) \left[\log \frac{q}{1-(1-q)^{d_{max}}} + (d-1)\log(1-q)\right] \\
&= N \log \frac{q}{1-(1-q)^{d_{max}}} + (M-N)\log(1-q),
\end{aligned}
$$

where $M = \sum d = 1^{max(d)} f(d)\, d$. Then, the log-likelihood function of Model 1 as a particular case of that of Model 2 in which $d_{max} = \infty$, i.e.

$$
\mathcal{L} = N \log q + (M-N)\log(1-q)
$$

since $q > 0$ and thus $\lim_{d_{max}\to\infty}(1-q)^{d_{max}} = 0$. For the *two-regime geometric models*, we start from the log-likelihood of Model 4, i.e.

$$
\begin{aligned}
\mathcal{L} &= \sum_{d=1}^{d^*} f(d) \log\left[c_1(1-q_1)^{d-1}\right] + \sum_{d=d^*+1}^{max(d)} f(d)\log\left[c_2(1-q_2)^{d-1}\right] \\
&= \sum_{d=1}^{d^*} f(d)\left[\log c_1 + (d-1)\log(1-q_1)\right] + \sum_{d=d^*+1}^{max(d)} f(d)\left[\log c_2 + (d-1)\log(1-q_2)\right] \\
&= N^* \log c_1 + (M^*-N^*)\log(1-q_1) + (N-N^*)\log c_2 + \\
&\quad (M-M^*-N+N^*)\log(1-q_2) \\
&= N^* \log c_1 + (N-N^*)\log c_2 + (M^*-N^*)\log\frac{1-q_1}{1-q_2} + (M-N)\log(1-q_2)
\end{aligned}
$$

where

$$
\begin{aligned}
M^* &= \sum_{d=1}^{d^*} f(d)\, d \\
N^* &= \sum_{d=1}^{d^*} f(d),
\end{aligned}
$$

while c_1 and c_2 are defined as explained in Section 2 for Model 3 and 4. Thus, the log-likelihood functions of Model 3 and Model 4 only differ in the computation of c_1 and c_2. For the *right truncated power-law* distribution, namely Model 5,

$$
\begin{aligned}
\mathcal{L} &= \sum_{d=1}^{max(d)} f(d)\log\frac{d^{-\gamma}}{H(d_{max},\gamma)} \\
&= \sum_{d=1}^{max(d)} f(d)\left[-\gamma \log d - \log H(d_{max},\gamma)\right] \\
&= -\gamma M' - N \log H(d_{max},\gamma),
\end{aligned}
$$

where $M' = \sum_{d=1}^{max(d)} f(d)\log(d)$. Finally, for Models 6 and 7, we start from the derivation of Model 7,

$$
\begin{aligned}
\mathcal{L} &= \sum_{d=1}^{d^*} f(d)\log(c_1 d^{-\gamma}) + \sum_{d=d^*+1}^{max(d)} f(d)\log\left[c_2(1-q)^{d-1}\right] \\
&= \sum_{d=1}^{d^*} f(d)\left[\log c_1 - \gamma\log(d)\right] + \sum_{d=d^*+1}^{max(d)} f(d)\left[\log c_2 + (d-1)\log(1-q)\right] \\
&= N^*\log c_1 - \gamma M'^* + (N - N^*)\log c_2 + (M - M^* - N + N^*)\log(1-q),
\end{aligned}
$$

while c_1 and c_2 are defined as explained in Section 2 for Model 6 and 7.

C Model selection validation

C.1 Artificial data generation

In the following, let $p_x(d)$ be the probability of d according to Model x. The parameter values used to generate each model are reported in Table 14, while sample size is $N = 10^4$ for each model. For right-truncated models sentence length is set to $n = 20$, and the maximum distance is set to $d_{max} = 19$. Then Model 0.0 is equivalent to Model 0 with $n = 20$. We choose $\gamma = 1.6$ because it has been obtained from fitting a right-truncated Zeta distribution to a Chinese treebank (Liu, 2007).

Table 14: Parameter values used to generate artificial samples. Here Model 0 is the same as Model 0.0.

Model	d_{max}	q	q_1	q_2	d^*	γ
0	19	-	-	-	-	-
1	-	0.2	-	-	-	-
2	19	0.2	-	-	-	-
3	-	-	0.5	0.1	4	-
4	19	-	0.5	0.1	4	-
5	19	-	-	-	-	1.6
6	-	0.2	-	-	4	1.6
7	19	0.2	-	-	4	1.6

Models 1 and 2 For the geometric distribution and its right-truncated version, namely Model 1 and Model 2, we use Dagpunar's fast inversion method (Dagpunar, 1988). For Model 1, a random distance d is obtained by producing a random uniform deviate x and then calculating

$$d = 1 + \left\lfloor \frac{\log x}{\lambda} \right\rfloor,$$

where $\lambda = \log(1-q)$, and q is the parameter of the desired geometric distribution. For Model 2, a value of d is produced until $d \leq d_{max}$.

Model 5 For Model 5, we employed the algorithm proposed by Devroye to efficiently generate a random deviate from a zeta distribution (Devroye, 1986), adapting it to allow for right-truncation. The algorithm is called one or more times until a value of d such that $d \leq d_{max}$ is obtained.

Model 0 and two-regime models For the sake of simplicity, random samples of Model 0 and of the two-regime models, namely Models 3, 4, 6, and 7, are generated using a tabular inversion method (Devroye, 1986; Muller, 1958). This method generates artificial distances in a pre-specified range, namely $d \in [1, \delta]$. Thus, in order to simulate Models 3 and 6 – which do not have a right-truncation – we set $\delta = 10^6$ to ensure that $p(d) \approx 0$ for $d \geq \delta$, while for Models 0, 4 and 7 we have $\delta = d_{max} = 19$. For simplicity, the method is implemented through binary search. Hence, a random deviate is produced in time $O(\log \delta)$.

C.2 Results

For each model, the best model yields a good visual fit to each artificially generated sample Figure 14. Indeed, the real underlying distribution is identified for every artificial random sample (Table 15 and Figure 13). See Figure 13 for the magnitude of the difference in BIC score between a given model and the best model (the model that minimizes BIC). The BIC of the double-regime models is always close to the BIC of the best model. The reason resides in the greater flexibility allowed by the existence of the break-point, which is however compensated by the penalty imposed on the additional parameter by the BIC score 9. Another concern could rise from the fitting of the random sample of Model 0, in which the BIC score of Model 4 is not much larger than that of the best model. Indeed, two geometric regimes could mimic the linearity of Model 0, but only in the case in which the second regime decays faster than the second. The values of the parameters estimated by maximum likelihood for each artificially generated random sample are shown in Table 16. See Table 17 for a comparison of the estimated values against the real values used to generate the data for each of the artificial samples. The error between the real values and the optimal parameters is either 0 or very small. In particular, maximum likelihood seems prone to underestimate the real value rather than the opposite.

Table 15: BIC scores on artificial random samples. Each row corresponds to a random sample generated by a given model. In each row, we show first the name of the true model and then we show the AIC values of each candidate model. The true Model 0 is Model 0 that is equivalent to Model 0.1 here. The candidate Model 0 is Model 0.0.

True model	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
0	55570.65	57527.90	55881.07	55750.98	55615.79	56724.85	55963.35	55697.17
1	60974.42	50037.40	50040.08	50049.99	50056.13	53256.86	50054.30	50057.24
2	51569.46	48995.12	48739.88	48801.18	48755.09	50086.11	48993.61	48757.98
3	76657.57	54995.13	55004.33	51553.49	51561.32	52694.62	51681.76	51689.79
4	51638.78	47122.05	46967.51	45359.06	44595.90	44716.30	44818.89	44685.95
5	49460.37	39609.04	39602.60	37251.07	37076.27	36864.76	36937.93	36881.25
6	61658.20	39436.89	39446.10	37196.76	37204.83	37684.75	37133.38	37141.30
7	48909.08	38217.08	38217.08	36343.48	36242.39	36270.85	36239.71	36175.27

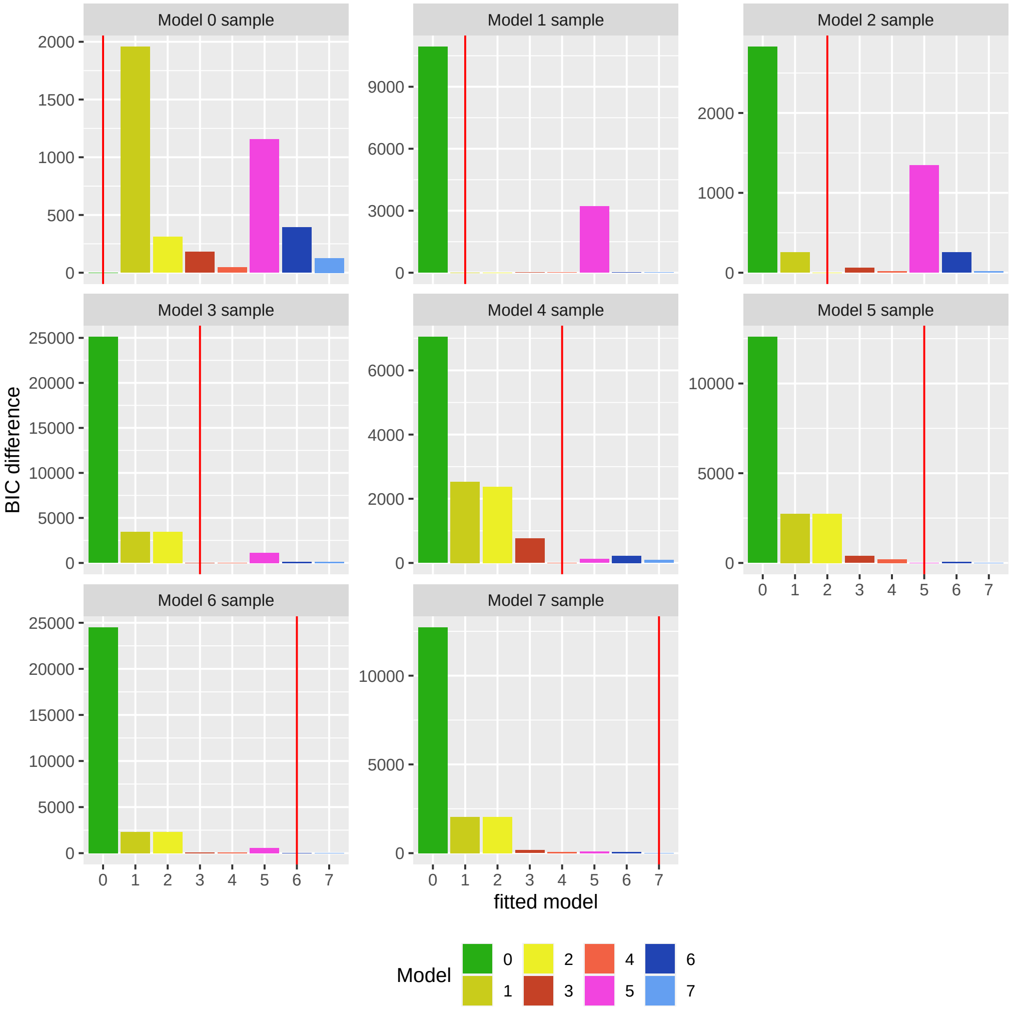


Figure 13: BIC differences in artificial random samples. The BIC difference is the difference between the BIC of the model and the BIC of the best model (the model that minimizes the BIC for the sample). The red vertical line indicates the best model according to BIC.

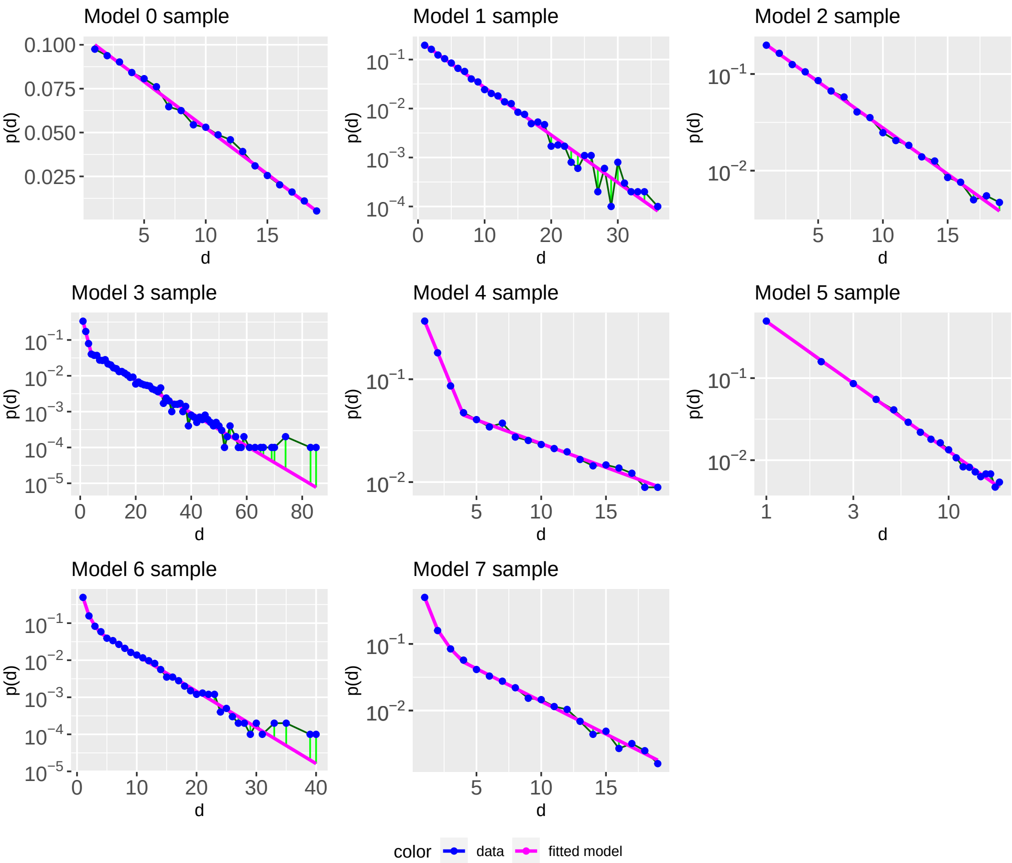


Figure 14: $p(d)$, the probability that a dependency link is formed between words at distance d according to the best model for artificially generated samples.

D Model selection results

We here report the results of model selection when sentences of any length are mixed for each language. See Table 18 and Table 20 for the AIC scores for PUD and PSUD, respectively; see Table 19 and Table 21 for the corresponding AIC differences. The AIC difference of a model is defined as the difference of its AIC and the AIC of the best model (the model that minimizes AIC) (Anderson and Burnham, 2004). The parameters estimated by maximum likelihood are shown in Table 22 for PUD and in Table 23 for PSUD. Finally, see Figure 15 for the best model fitted to the empirical distribution for languages in PSUD.

Table 18: AIC scores of each model in the PUD collection on sentences of mixed lengths. Here Model 0 refers to Model 0.1.

Language	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Arabic	84577	55188	55190	52011	52012	52264	51866	51864
Chinese	86281	68121	68123	65826	65827	67025	65737	65738
Czech	68424	48729	48731	47872	47872	49499	48212	48214
English	85821	60122	60124	59402	59402	62649	60055	60056
Finnish	52893	38423	38425	37955	37956	38921	37927	37928
French	109197	71748	71750	69420	69418	72291	70944	70946
German	86626	68510	68512	66699	66700	68821	66955	66957
Hindi	107388	83075	83077	75832	75828	76676	75788	75782
Icelandic	73752	50242	50244	49411	49413	51252	49716	49718
Indonesian	76351	50676	50678	48875	48876	49596	48916	48917
Italian	104223	68313	68315	66370	66369	69289	67786	67788
Japanese	135512	93746	93748	85222	85221	87112	86524	86525
Korean	64173	50365	50367	45474	45472	45647	45337	45332
Polish	66255	45103	45105	43851	43852	44719	43956	43958
Portuguese	100042	67213	67215	65361	65361	68010	66557	66559
Russian	70474	47879	47881	46750	46751	48291	47201	47203
Spanish	101194	67353	67355	65377	65376	67934	66641	66643
Swedish	75639	53807	53809	53135	53136	55623	53633	53635
Thai	108081	69553	69555	65717	65718	66242	65519	65521
Turkish	62864	51439	51441	47362	47358	47697	47250	47245

Table 19: AIC differences of each model in the PUD collection on sentences of mixed lengths. Here Model 0 refers to Model 0.1.

Language	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Arabic	32712.96	3323.95	3325.95	146.57	147.28	399.60	1.44	0.00
Chinese	20544.43	2383.68	2385.68	88.77	90.07	1288.62	0.00	0.72
Czech	20552.34	857.52	859.51	0.00	0.70	1627.44	340.07	341.97
English	26419.25	720.65	722.64	0.00	0.63	3247.77	652.91	654.90
Finnish	14965.50	496.10	498.00	27.96	29.28	993.94	0.00	1.02
French	39779.35	2329.86	2331.86	1.94	0.00	2873.31	1526.42	1528.33
German	19927.31	1811.19	1813.19	0.00	1.49	2122.61	256.63	258.34
Hindi	31605.72	7292.04	7294.03	49.53	45.49	893.65	5.61	0.00
Icelandic	24340.78	831.13	833.13	0.00	1.94	1841.66	305.52	307.52
Indonesian	27476.04	1800.38	1802.38	0.00	1.08	721.10	41.17	41.86
Italian	37854.79	1944.06	1946.06	1.10	0.00	2920.54	1417.74	1419.67
Japanese	50290.71	8524.56	8526.56	0.58	0.00	1890.84	1302.96	1303.51
Korean	18840.12	5032.98	5034.98	141.29	139.91	314.13	4.31	0.00
Polish	22403.20	1251.25	1253.25	0.00	0.73	867.34	104.69	106.27
Portuguese	34681.44	1852.22	1854.22	0.00	0.34	2649.16	1196.46	1198.33
Russian	23723.74	1129.28	1131.28	0.00	1.37	1540.80	451.36	453.32
Spanish	35817.93	1976.75	1978.74	1.07	0.00	2557.41	1264.87	1266.59
Swedish	22503.61	671.54	673.54	0.00	0.88	2487.59	497.78	499.75
Thai	42561.20	4033.29	4035.29	197.05	198.82	722.50	0.00	1.23
Turkish	15618.79	4193.97	4195.96	117.05	112.74	452.16	5.51	0.00

Table 20: AIC scores of each model in the PSUD collection on sentences of mixed lengths. Here Model 0 refers to Model 0.1.

Language	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Arabic	83964	49718	49720	45461	45461	45444	45249	45248
Chinese	86004	66658	66660	63187	63188	63852	63043	63043
Czech	67743	43660	43662	42048	42049	42711	42164	42166
English	84875	51868	51870	49860	49860	50895	50293	50295
Finnish	52389	35247	35249	34450	34451	35057	34434	34436
French	108313	62309	62311	57458	57458	58294	58037	58036
German	85580	64289	64291	62110	62111	63642	62229	62230
Hindi	106846	79001	79003	68777	68760	69540	68495	68483
Icelandic	72807	42153	42155	39927	39929	40396	40000	40002
Indonesian	75765	45212	45214	41927	41928	42005	41809	41808
Italian	103370	59445	59447	55354	55354	56373	56039	56040
Japanese	135293	88560	88562	72667	72667	72316	71831	71831
Korean	64065	49797	49799	44509	44508	44683	44366	44364
Polish	65708	40765	40767	38674	38676	39020	38697	38698
Portuguese	99155	58440	58442	54381	54381	55150	54846	54846
Russian	69999	43597	43599	41455	41457	41963	41541	41543
Spanish	100350	58907	58909	54617	54615	55352	55105	55103
Swedish	74683	46288	46290	44584	44586	45399	44815	44817
Thai	107549	63835	63837	57879	57881	57879	57564	57565
Turkish	62784	50771	50773	46448	46442	46728	46325	46318

Table 21: AIC differences of each model in the PSUD collection on sentences of mixed lengths. Here Model 0 refers to Model 0.1.

Language	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Arabic	38715.46	4469.92	4471.92	212.61	213.36	196.35	0.49	0.00
Chinese	22961.93	3615.17	3617.16	144.40	145.21	809.83	0.67	0.00
Czech	25695.55	1612.08	1614.08	0.00	1.23	663.54	116.10	117.76
English	35015.25	2008.17	2010.17	0.09	0.00	1034.96	433.57	435.32
Finnish	17954.96	812.85	814.84	15.62	16.93	622.93	0.00	1.41
French	50855.16	4851.44	4853.44	0.05	0.00	836.35	579.48	578.26
German	23469.70	2179.07	2181.07	0.00	1.12	1532.51	118.75	120.29
Hindi	38363.24	10518.04	10520.03	294.32	277.30	1056.95	11.70	0.00
Icelandic	32879.81	2225.42	2227.42	0.00	1.79	468.28	72.30	74.23
Indonesian	33956.92	3404.16	3406.16	119.39	119.95	196.85	1.41	0.00
Italian	48016.12	4091.05	4093.05	0.00	0.83	1019.71	685.68	686.27
Japanese	63462.28	16729.23	16731.23	836.09	836.29	485.06	0.28	0.00
Korean	19701.40	5432.90	5434.90	145.71	144.65	319.57	1.91	0.00
Polish	27034.25	2090.57	2092.57	0.00	1.52	346.04	23.00	23.88
Portuguese	44774.69	4059.49	4061.49	0.00	0.29	769.23	465.71	465.70
Russian	28543.92	2141.67	2143.67	0.00	1.47	508.13	86.14	87.86
Spanish	45735.15	4292.48	4294.48	2.28	0.00	736.75	490.45	487.74
Swedish	30099.05	1703.55	1705.55	0.00	1.64	815.39	231.31	233.16
Thai	49985.45	6271.66	6273.66	315.48	317.16	315.43	0.00	1.29
Turkish	16466.09	4453.49	4455.47	129.96	124.00	410.41	7.45	0.00

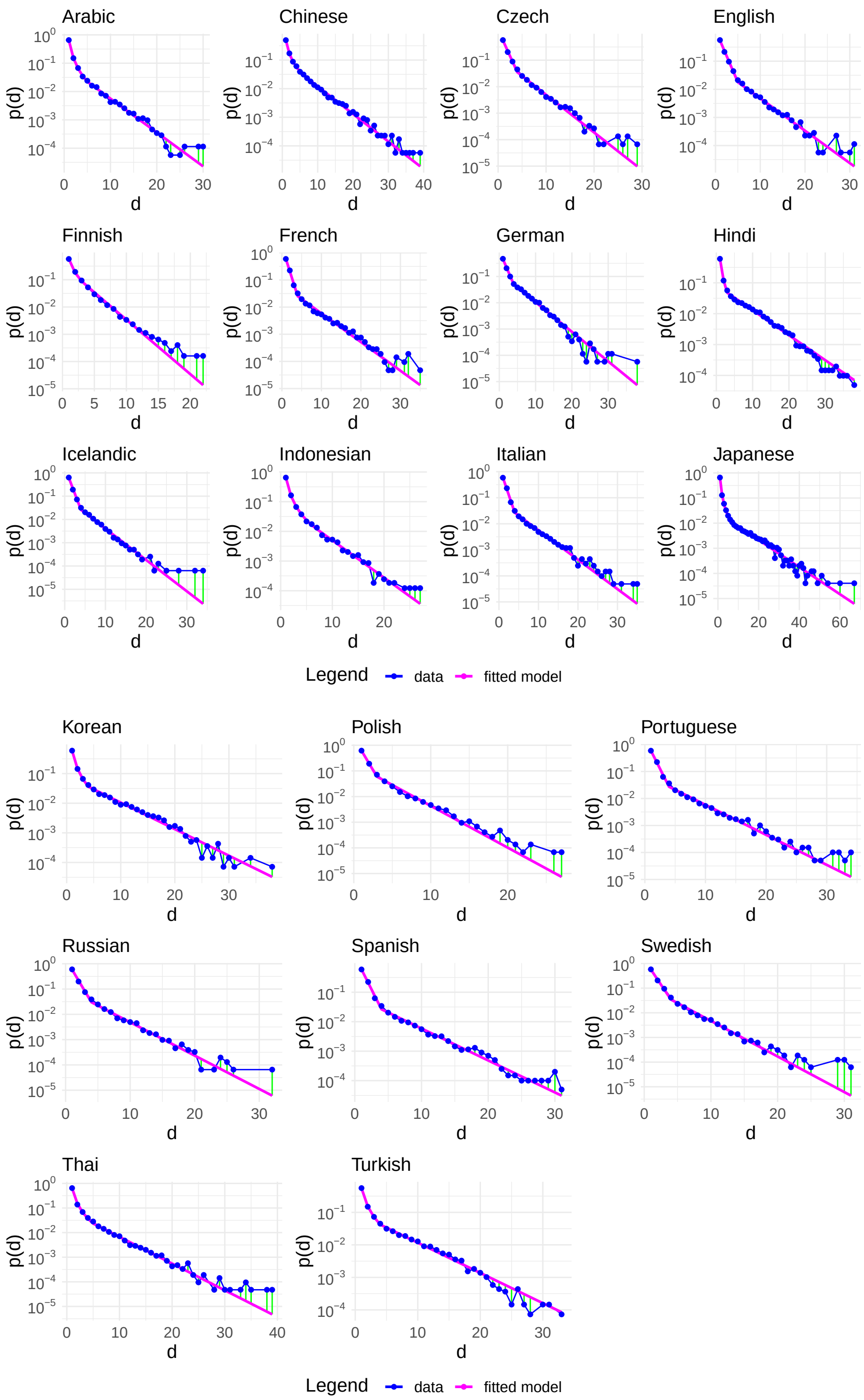
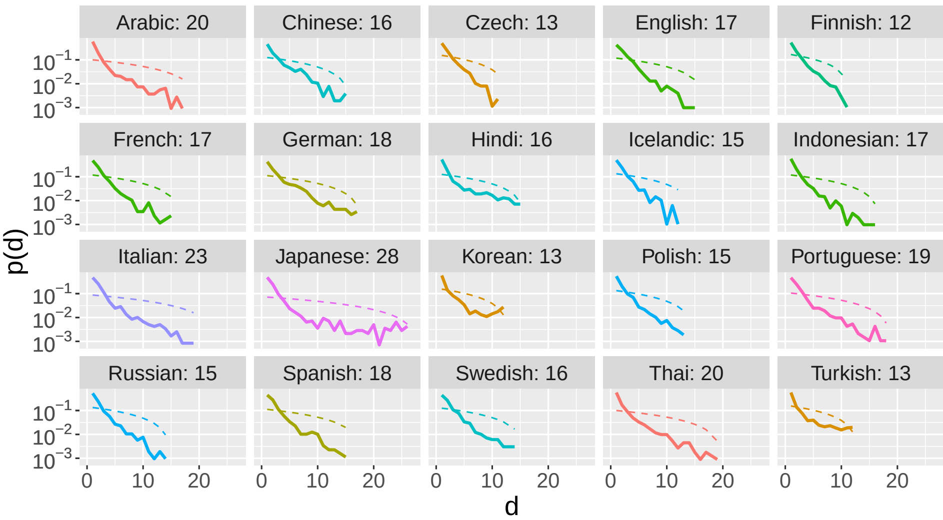


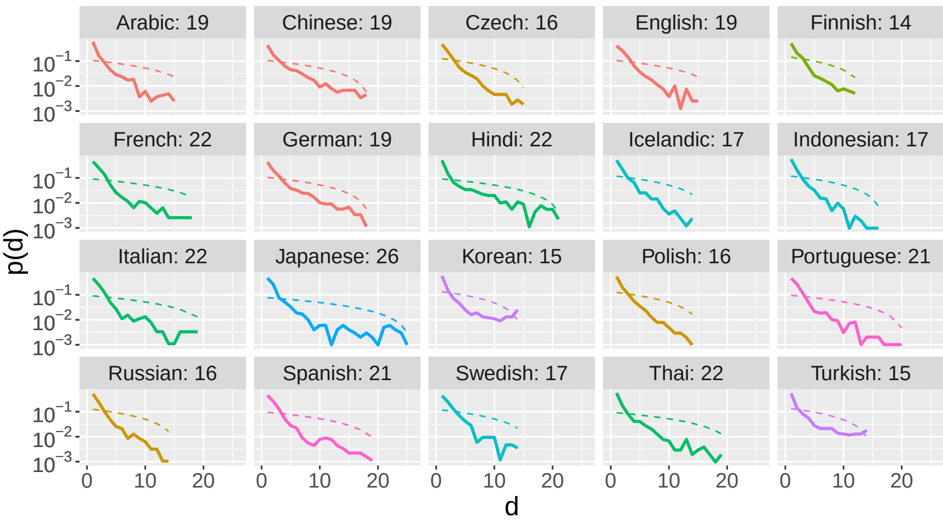
Figure 15: $p(d)$, the probability that a dependency link is formed between words at distance d according to the data and the best model for every language in PSUD.

E The distribution of dependency distances for characteristic sentence lengths.

See Figure 16 (a-b) for the distributions in PUD, for modal and mean sentence length respectively; see Figure 17 (a-b) for PSUD. As mean sentence length, we use the results of rounding the actual mean sentence length to the nearest integer.

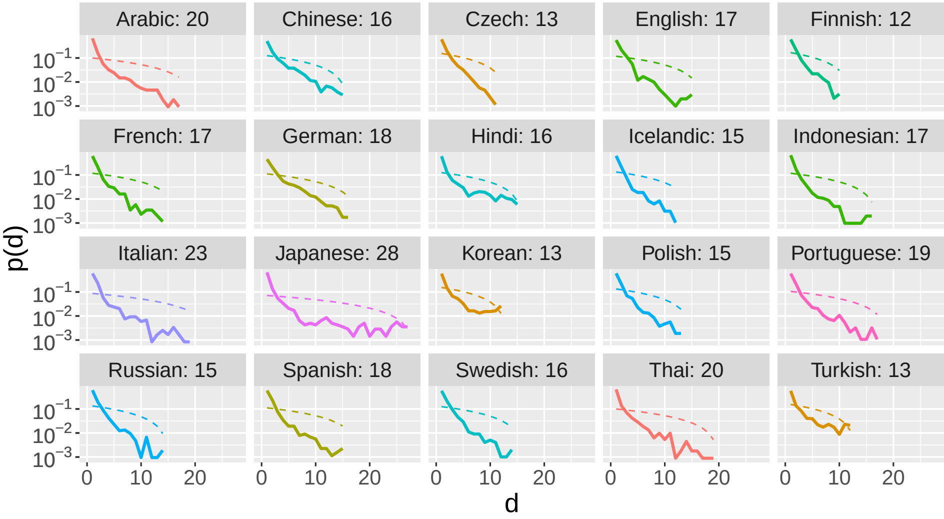


(a) Modal sentence length.

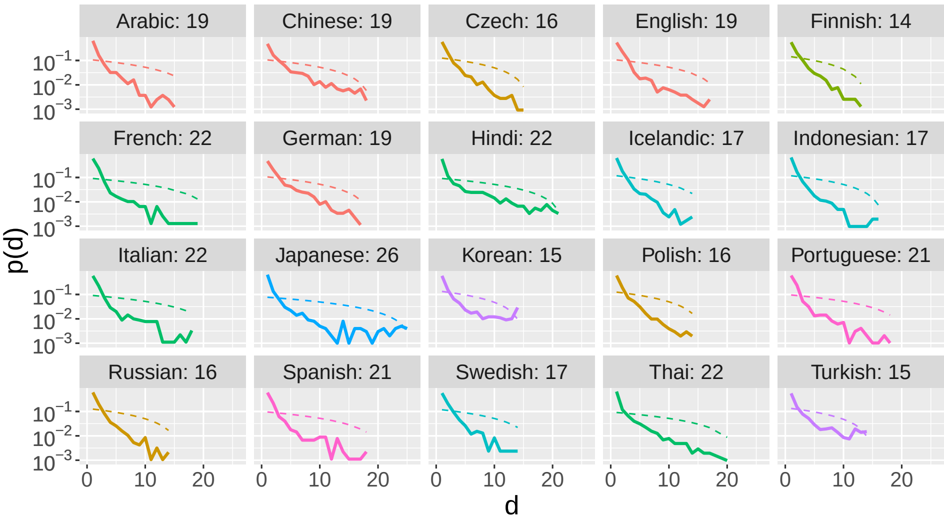


(b) Mean sentence length.

Figure 16: $p(d)$, the probability that linked words are at distance d in sentences of modal (a) and mean (b) length for each language in PUD. Mode and mean are shown next to the respective language label. The dashed line shows the probability according to Model 0 (1). Points where $p(d) = 0$ are not shown.



(a) Modal sentence length.



(b) Mean sentence length.

Figure 17: $p(d)$, the probability that linked words are at distance d in sentences of modal (a) and mean (b) length for each language in PSUD. The format is the same as in Figure 16.

Table 13: The log-likelihood $\mathcal{L}$ for each of the probability mass functions. K is the number of free parameters, N is the sample size, M is the sum of distances weighted by frequency, i.e. $M = \sum_{i=1}^{max(d)} f(d_i)d_i$, M^* is the same sum up to d^*, i.e. $M^* = \sum_{i=1}^{d^*} f(d_i)d_i$, M' is the sum of log distances weighted by frequency, i.e. $M' = \sum_{i=1}^{max(d)} f(d_i)log(d_i)$, and W is such that $W = \sum_{i=1}^{max(d)} f(d_i)log(d_{max}+1-d_i)$. N^* is the sum of distance frequencies up to d^*, i.e. $N^* = \sum_{i=1}^{d^*} f(d_i)$, and M'^* is M' up to d^*, i.e. $M'^* = \sum_{i=1}^{d^*} f(d_i)log(d_i)$. For model 3, c_1 id defined in 3; for model 4, c_1 is defined in 5; for models 3 and 4, $c_2 = \tau c_1$ with τ defined as in 4. For model 6, c_1 id defined in 6; for model 7, c_1 is defined in 8; for models 6 and 7, $c_2 = \tau c_1$ with τ defined as in 7. Finally, $W_n = \sum_{i=1}^{max(d)} f(d_i)\log(n-d_i))$ and $N_n = \sum_{i=1}^{max(d)} f(d_i)$ in sentences of length n.

Model	**Function**	K	$\mathcal{L}$
0.0	Null model	1	$N\log(\frac{2}{d_{max}(d_{max}+1)}) + W$
0.1	Extended Null model	0	$\sum_{n=min(n)}^{max(n)}[N_n \log\left(\frac{2}{n(n-1)}\right) + W_n]$
1	Geometric	1	$N\log q + (M-N)\log(1-q)$
2	Right-truncated geometric	2	$N\log\left(\frac{q}{1-(1-q)^{d_{max}}}\right) + (M-N)\log(1-q)$
3	Two-regime geometric	3	$N^*\log c_1 + (N-N^*)\log c_2 + (M^*-N^*)\log\left(\frac{1-q_1}{1-q_2}\right) + (M-N)\log(1-q_2)$
4	Two-regime right-truncated geometric	4	$N^*\log c_1 + (N-N^*)\log c_2 + (M^*-N^*)\log\left(\frac{1-q_1}{1-q_2}\right) + (M-N)\log(1-q_2)$
5	Right-truncated zeta distribution	2	$-\gamma M' - N\log(H(d_{max},\gamma))$
6	Two-regime zeta-geometric	3	$N^*log(c_1) - \gamma M'^* + (N-N^*)log(c_2) + (M-M^*-N+N^*)log(1-q)$
7	Two-regime right-truncated zeta-geometric	4	$N^*log(c_1) - \gamma M'^* + (N-N^*)log(c_2) + (M-M^*-N+N^*)log(1-q)$

Table 16: Best parameters estimated in artificial random samples by maximum likelihood. The 1st column indicates the true model while the header row indicates the candidate model. Here Model 0 refers to Model 0.0.

Model	$\max(d)$	d_{max}	q	q	d_{max}	q_1	q_2	d^*	q_1	q_2	d^*	d_{max}	d_{max}	γ	γ	q	d^*	γ	q	d^*	d_{max}
		0	1	2		3			4				5		6			7			
0	19	19	0.142	0.100	19	0.088	0.643	17	0.071	0.242	13	19	19	0.522	0.302	0.343	13	0.264	0.206	11	19
1	36	36	0.200	0.200	36	0.200	0.543	34	0.191	0.203	5	36	36	1.204	0.274	0.201	2	0.278	0.201	2	36
2	19	19	0.210	0.197	19	0.197	0.622	18	0.199	0.091	17	19	19	0.985	0.418	0.221	4	0.295	0.198	2	19
3	85	85	0.160	0.160	85	0.502	0.101	4	0.502	0.101	4	85	85	1.422	1.373	0.103	5	1.373	0.102	5	85
4	19	19	0.228	0.219	19	0.549	0.166	3	0.503	0.101	4	19	19	1.242	1.234	0.644	18	1.332	0.097	6	19
5	19	19	0.314	0.313	19	0.628	0.201	3	0.641	0.180	3	19	19	1.582	1.578	0.610	18	1.588	0.058	15	19
6	40	40	0.317	0.317	40	0.623	0.204	3	0.624	0.204	3	40	40	1.718	1.613	0.201	4	1.614	0.201	4	40
7	19	19	0.333	0.332	19	0.613	0.221	3	0.622	0.206	3	19	19	1.608	1.541	0.299	12	1.610	0.202	4	19

Table 17: Best estimated parameters, real parameters used to generate the artificial samples, and their difference. Here Model 0 refers to Model 0.0. The header row indicates the true model.

	0	1	2		3			4				5		6			7			
	d_{max}	q	q	d_{max}	q_1	q_2	d^*	q_1	q_2	d^*	d_{max}	d_{max}	γ	γ	q	d^*	γ	q	d^*	d_{max}
estimated	19	0.200	0.197	19	0.502	0.101	4	0.503	0.101	4	19	19	1.582	1.613	0.201	4	1.610	0.202	4	19
real	19	0.200	0.200	19	0.500	0.100	4	0.500	0.100	4	19	19	1.600	1.600	0.200	4	1.600	0.200	4	19
error	0	0.000	−0.003	0	0.002	0.001	0	0.003	0.001	0	0	0	−0.018	0.013	0.001	0	0.010	0.002	0	0

Table 22: Best parameters estimated by maximum likelihood on sentences of mixed lengths in the PUD collection.

			1	2		3			4				5		6			7			
Language	$\max(n)$	$\max(d)$	q	q	d_{max}	q_1	q_2	d^*	q_1	q_2	d^*	d_{max}	d_{max}	γ	γ	q	d^*	γ	q	d^*	d_{max}
Arabic	50	30	0.434	0.434	30	0.668	0.269	3	0.668	0.269	3	30	30	1.973	1.840	0.240	7	1.842	0.238	7	30
Czech	44	29	0.418	0.418	29	0.499	0.270	5	0.500	0.269	5	29	29	1.837	1.385	0.347	3	1.385	0.347	3	29
German	50	42	0.322	0.322	42	0.485	0.222	4	0.485	0.222	4	42	42	1.675	1.351	0.234	5	1.351	0.234	5	42
English	56	31	0.395	0.395	31	0.453	0.256	6	0.454	0.255	6	31	31	1.759	0.930	0.380	2	0.930	0.380	2	31
Finnish	39	21	0.446	0.446	21	0.639	0.388	2	0.562	0.360	3	21	21	1.855	1.440	0.374	3	1.440	0.374	3	21
French	54	36	0.396	0.396	36	0.491	0.197	6	0.492	0.196	6	36	36	1.826	1.462	0.296	4	1.462	0.296	4	36
Hindi	58	42	0.303	0.303	42	0.671	0.175	3	0.672	0.174	3	42	42	1.735	1.798	0.170	4	1.800	0.169	4	42
Indonesian	47	27	0.442	0.442	27	0.629	0.305	3	0.630	0.304	3	27	27	1.935	1.713	0.295	5	1.714	0.294	5	27
Icelandic	52	34	0.432	0.432	34	0.531	0.309	4	0.531	0.309	4	34	34	1.888	1.412	0.359	3	1.412	0.359	3	34
Italian	60	35	0.403	0.403	35	0.490	0.207	6	0.491	0.206	6	35	35	1.830	1.446	0.307	4	1.446	0.307	4	35
Japanese	70	65	0.337	0.337	65	0.521	0.119	6	0.522	0.118	6	65	65	1.849	1.754	0.130	13	1.755	0.130	13	65
Korean	43	37	0.364	0.364	37	0.700	0.197	3	0.701	0.197	3	37	37	1.886	1.886	0.180	5	1.888	0.179	5	37
Polish	39	27	0.449	0.449	27	0.569	0.288	4	0.569	0.287	4	27	27	1.936	1.653	0.324	4	1.653	0.324	4	27
Portuguese	58	34	0.396	0.396	34	0.504	0.234	5	0.504	0.233	5	34	34	1.812	1.436	0.301	4	1.436	0.301	4	34
Russian	47	32	0.440	0.440	32	0.529	0.261	5	0.529	0.260	5	32	32	1.916	1.564	0.332	4	1.564	0.332	4	32
Spanish	58	32	0.399	0.399	32	0.510	0.231	5	0.510	0.230	5	32	32	1.816	1.460	0.300	4	1.460	0.300	4	32
Swedish	49	31	0.404	0.404	31	0.462	0.257	6	0.462	0.257	6	31	31	1.791	1.214	0.358	3	1.214	0.358	3	31
Thai	63	38	0.409	0.409	38	0.653	0.258	3	0.653	0.258	3	38	38	1.933	1.770	0.230	7	1.770	0.230	7	38
Turkish	37	34	0.343	0.343	34	0.670	0.201	3	0.671	0.200	3	34	34	1.797	1.812	0.195	4	1.815	0.194	4	34
Chinese	49	39	0.323	0.323	39	0.569	0.233	3	0.569	0.232	3	39	39	1.694	1.439	0.219	6	1.440	0.219	6	39

Table 23: Best parameters estimated by maximum likelihood on sentences of mixed lengths in the PSUD collection.

			1	2		3			4				5		6			7			
Language	$\max(d)$	$\max(n)$	q	q	d_{max}	q_1	q_2	d^*	q_1	q_2	d^*	d_{max}	d_{max}	γ	γ	q	d^*	γ	q	d^*	d_{max}
Arabic	50	30	0.488	0.488	30	0.727	0.263	3	0.727	0.263	3	30	30	2.150	2.101	0.241	5	2.102	0.241	5	30
Czech	44	29	0.475	0.475	29	0.602	0.279	4	0.602	0.279	4	29	29	2.027	1.814	0.306	5	1.814	0.306	5	29
German	50	38	0.355	0.355	38	0.523	0.228	4	0.523	0.228	4	38	38	1.758	1.489	0.244	5	1.489	0.244	5	38
English	56	31	0.472	0.472	31	0.575	0.234	5	0.575	0.233	5	31	31	2.025	1.810	0.299	5	1.810	0.299	5	31
Finnish	39	22	0.490	0.490	22	0.625	0.362	3	0.625	0.362	3	22	22	2.007	1.715	0.369	4	1.715	0.369	4	22
French	54	35	0.470	0.470	35	0.652	0.216	4	0.652	0.216	4	35	35	2.090	2.007	0.206	9	2.008	0.204	9	35
Hindi	58	38	0.329	0.329	38	0.729	0.162	3	0.730	0.161	3	38	38	1.843	2.298	0.171	3	2.302	0.170	3	38
Indonesian	47	27	0.500	0.500	27	0.717	0.283	3	0.717	0.283	3	27	27	2.150	2.055	0.254	7	2.056	0.251	7	27
Icelandic	52	34	0.521	0.521	34	0.653	0.266	4	0.653	0.266	4	34	34	2.184	2.029	0.295	6	2.030	0.295	6	34
Italian	60	35	0.475	0.475	35	0.645	0.229	4	0.645	0.228	4	35	35	2.087	1.981	0.226	8	1.982	0.225	8	35
Japanese	70	67	0.367	0.367	67	0.699	0.121	4	0.699	0.121	4	67	67	2.060	2.218	0.117	6	2.218	0.117	6	67
Korean	43	38	0.370	0.370	38	0.713	0.193	3	0.713	0.193	3	38	38	1.915	2.016	0.187	4	2.018	0.186	4	38
Polish	39	27	0.501	0.501	27	0.688	0.313	3	0.688	0.313	3	27	27	2.112	1.970	0.287	6	1.971	0.286	6	27
Portuguese	58	34	0.469	0.469	34	0.642	0.226	4	0.642	0.225	4	34	34	2.074	1.971	0.224	8	1.972	0.223	8	34
Russian	47	32	0.489	0.489	32	0.629	0.262	4	0.629	0.262	4	32	32	2.092	1.930	0.282	6	1.930	0.282	6	32
Spanish	58	31	0.469	0.469	31	0.646	0.222	4	0.647	0.221	4	31	31	2.071	1.981	0.221	8	1.982	0.219	8	31
Swedish	49	31	0.482	0.482	31	0.607	0.283	4	0.607	0.283	4	31	31	2.050	1.832	0.309	5	1.832	0.309	5	31
Thai	63	39	0.454	0.454	39	0.723	0.240	3	0.723	0.240	3	39	39	2.100	2.053	0.221	5	2.053	0.221	5	39
Turkish	37	33	0.350	0.350	33	0.680	0.199	3	0.681	0.198	3	33	33	1.818	1.859	0.194	4	1.863	0.193	4	33
Chinese	49	39	0.335	0.335	39	0.619	0.219	3	0.619	0.219	3	39	39	1.755	1.574	0.200	7	1.574	0.199	7	39